

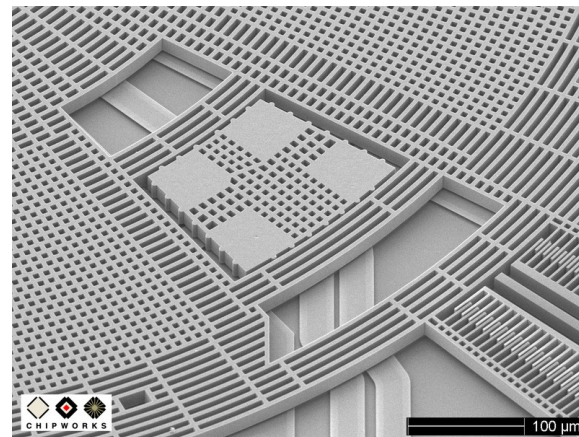
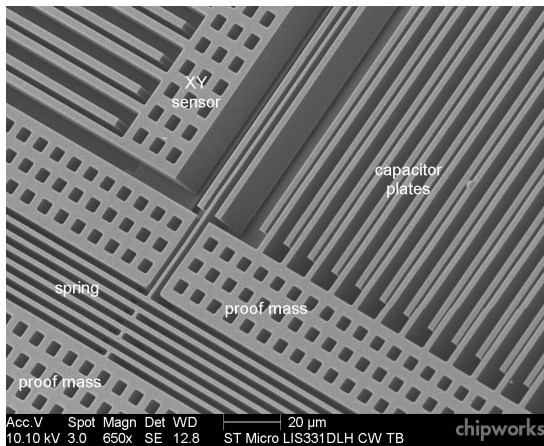
ELECTROSTATIC scaling in MEMS actuators

Scaling in Electrostatics

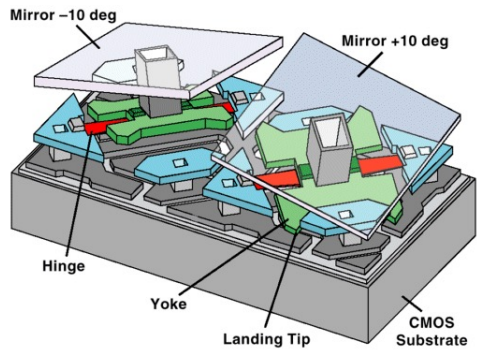
- Parallel plate Capacitor
- Energy density in capacitors, Paschen curve
- Parallel plate actuator, pull-in, spring softening
- Zipping actuators
- Comb drive

Concepts to master - Electrostatics

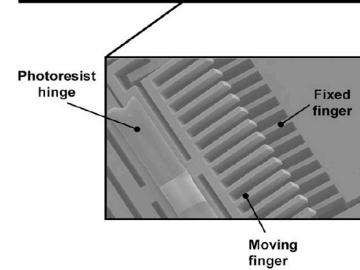
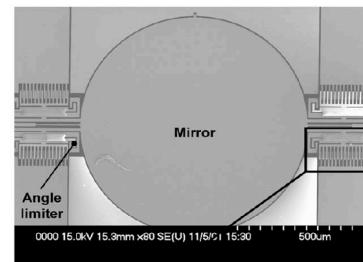
- Capacitive actuator
 - Energy density in a capacitor (parallel plate and comb)
 - Force derived from energy
 - Scaling of Force with geometry
 - Spring constant derived from force
 - Spring softening (parallel plate vs. comb drive)
 - Pull-in instability in parallel plate
 - Failure mode comb-drive
 - Paschen curve in air and implications for scaling
 - Resonators
 - Drive and sense principles
 - Temperature drift and solutions



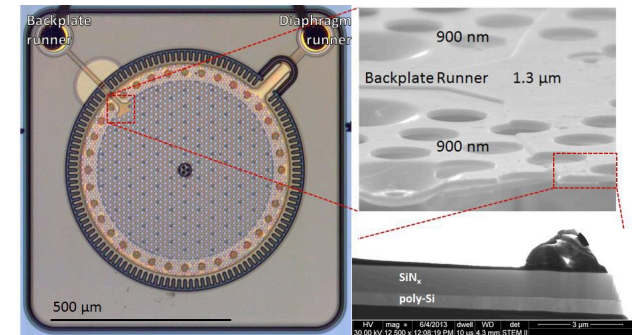
Inertial sensing: accelerometer and gyro (photos of ST micro devices)



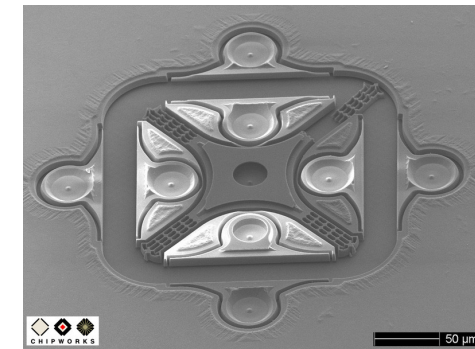
Displays (TI DMD, Qualcomm Mirasol)



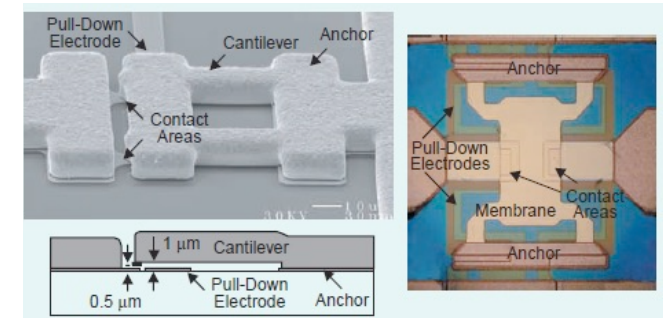
MEMS optical scanner
(Hah et al, 2004) IEEE Journal of
Selected Topics in Quantum
Electronics



Microphone
M. Broas, et al, 2015 (ECTC)



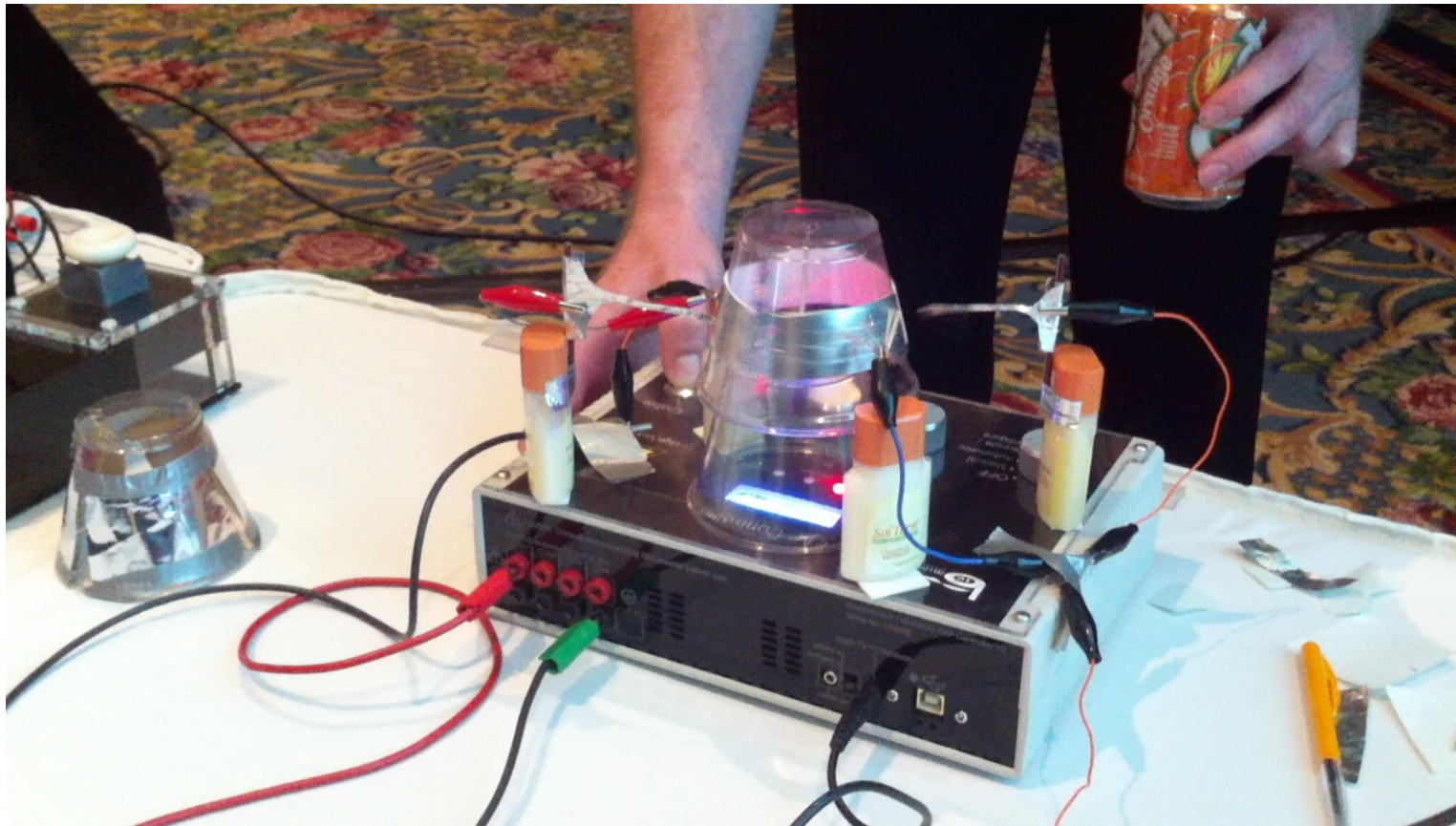
RF resonator (SiTime)



RF switch (Analog devices, Raytheon)

Electrostatic MEMS are everywhere!

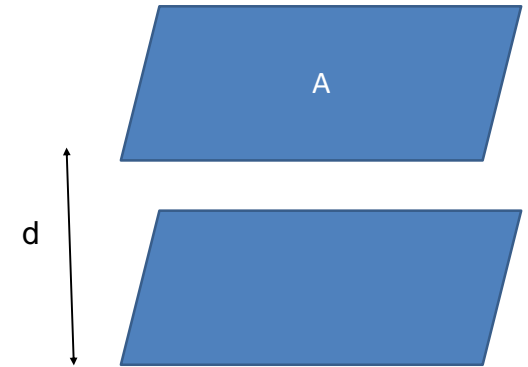
cm-scale electrostatic motor ! (made from parts stolen from a motel bathroom, plus a HV supply)



Biomimetic lab, U. Auckland, NZ
O'Brien et al., . "Rotating turkeys and self-commutating artificial muscle motors."
Applied Physics Letters 100, no. 7 (2012)

Capacitor scaling

- Capacitance (parallel plate) $C = \frac{\epsilon\epsilon_0 A}{d} \propto L$
- Charge $Q = CV = \epsilon\epsilon_0 \frac{A}{d} V$
- For $V=\text{constant}$ $Q \propto \frac{A}{d}$ but for $E=\text{constant}$ $Q \propto A$
- in good insulators $E_{\text{breakdown}} = 1 \text{ to } 3 \text{ V/nm}$, but $E_{\text{breakdown}}$ can be 1000x lower in air
- A (surface area) is often the only effective dimensional parameter because d is limited by E_{BD}



C: capacitance
 A: area
 d: gap
 V: voltage
 E: electric field
 E_{BD} : breakdown electric field
 Q: charge
 ϵ_0 : permittivity of free space
 ϵ or ϵ_r : relative permittivity

Voltage fluctuations in a capacitor

$$\overline{E_{th}} = \frac{1}{2} k_B T = \frac{1}{2} C v_n^2 \quad \text{Equi-partition theorem, like for noise in a resistor or in mechanical systems}$$

$$v_{nrms} = \sqrt{\frac{k_b T}{C}} \quad v_{nrms} = \sqrt{\frac{k_b T}{\epsilon \epsilon_0}} \sqrt{\frac{d}{A}} \propto L^{-1/2} \quad \Delta Q_{rms} = C \cdot v_{n,rms} = \sqrt{k_b T C}$$



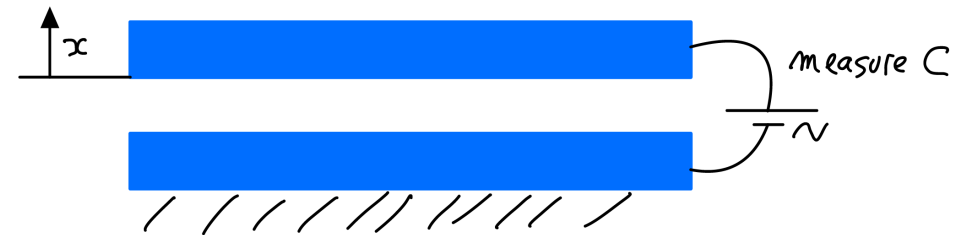
At room temperature (300K) $k_B T = 4.14 \cdot 10^{-21} \text{ J}$

for discrete capacitor $C=20 \text{ pF}$ $v_{rms} = 14 \text{ } \mu\text{V}$ (i.e. 1000 e)

for $1 \text{ } \mu\text{m}^2$, $0.1 \text{ } \mu\text{m}$ dielectric ($\epsilon=2$) $C=0.16 \text{ fF}$, $v_{rms} = 5 \text{ mV}$, (i.e. 5e)

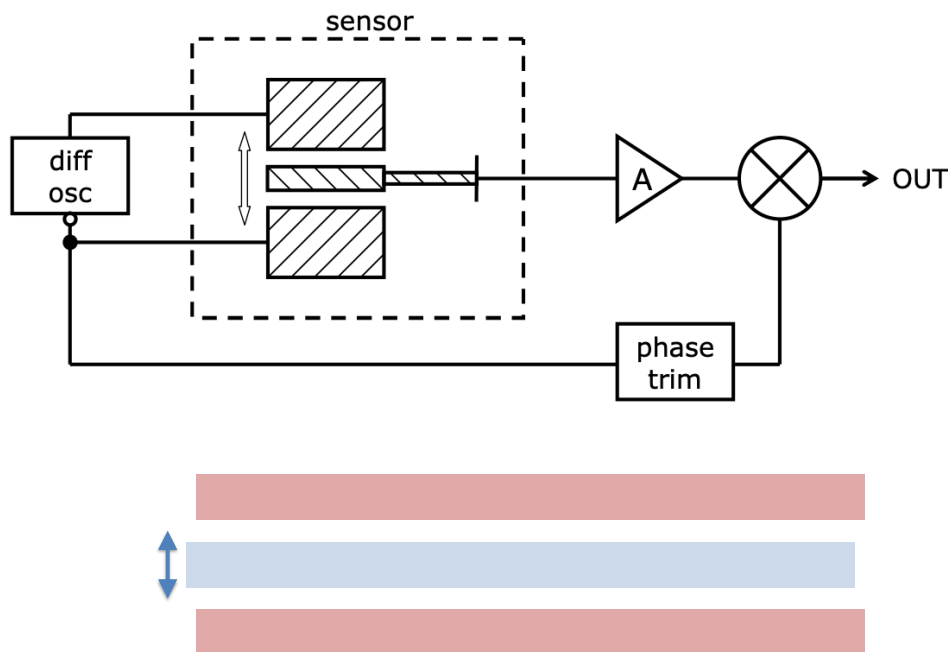
Capacitance sensing

- Parallel plate capacitance with displacement x $C = \frac{\epsilon_0 A}{d + x}$
- Sensitivity $S_0 = \frac{dC}{dx} = -\frac{C}{d}$ $S_0 \propto \frac{A}{d^2} \propto L^0$
- Must scale down d to maintain S_0 if one reduces A
- Limits to downscaling capacitive sensing:
 - voltage noise
 - in small gaps, bias measurement voltage must be decreased due to E-field limitation
=> decrease of voltage sensitivity
 - defects / inhomogeneities in small gaps
 - probe voltage induce electrostatic force that could provoke pull-in
 - electrostatic “spring constant” affects dynamical properties $k_{es} = \frac{dF_{es}}{dx}$

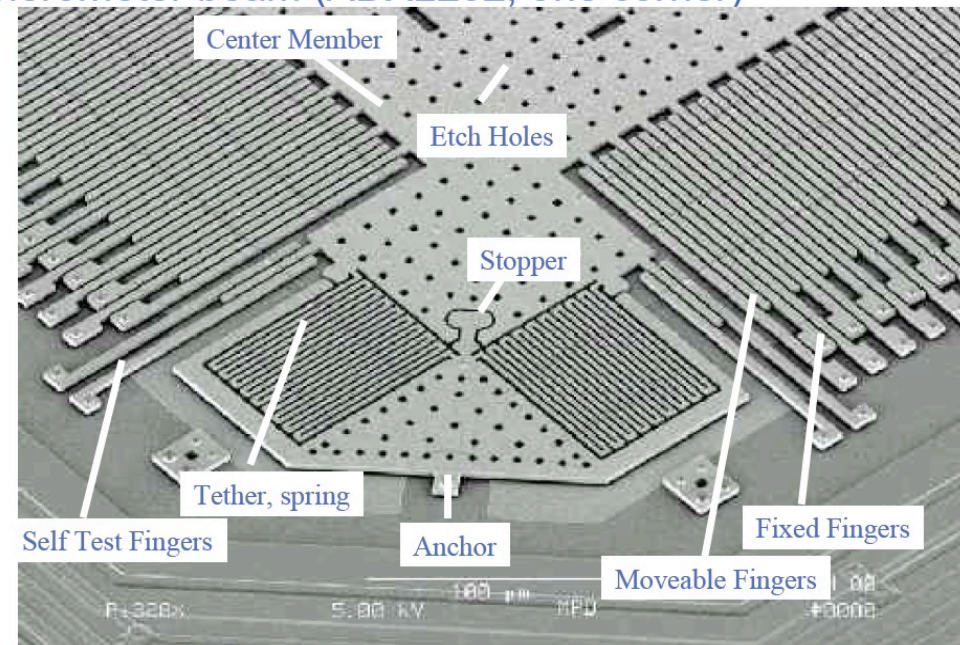


Capacitive sensing in MEMS

- Generally accomplished using a differential setup for accelerometers and gyros



Accelerometer beam (ADXL202, one corner)



(looks like a comb drive, but is not: fingers move perpendicular to their long axis, using many fingers to increase capacitance)

PARALLEL PLATE ELECTROSTATIC ACTUATORS

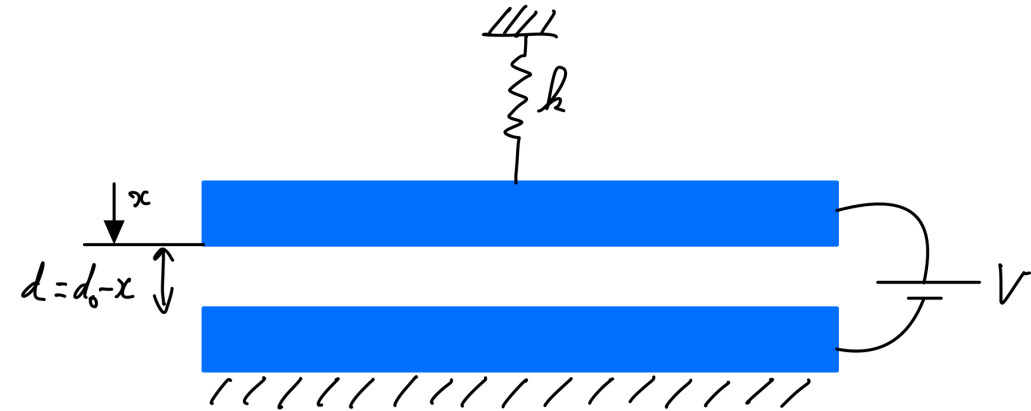
- Energy density, Paschen curve
- In-plane motion (constant gap)
- Closing gap motion
- Elastomer dielectric
- Zipping

Parallel plates electrostatic actuator

- Electrostatic energy in a capacitor $E_{es} = \frac{1}{2} CV^2$
- Normal electrostatic force for **fixed applied voltage V**:

$$F_{es} = \frac{dE_{es}}{dx} = \frac{1}{2} \frac{dC}{dx} V^2 = \frac{1}{2} \frac{CV^2}{d} \propto 1/d^2$$

$$F_{es} = \frac{\epsilon_0 A V^2}{2d^2} = \frac{\epsilon_0 A}{2} E^2$$



- Or for **fixed charge Q**

$$F_{es} = \frac{Q^2}{2\epsilon A} \quad (\text{F independent of } d \text{ for charge control!})$$

- Scaling of ES force:

- For constant Voltage (V indep of size): $F_{ES} \propto L^0 \propto \left(\frac{A}{d^2}\right)$
- For constant E field (V proportional to d): $F_{ES} \propto L^2$

ES Force is always attractive!

Parallel plate electrostatic actuator

- Normal electrostatic force for an applied voltage V :

$$F_{es} = \frac{\epsilon_0 A V^2}{2d^2} = \frac{\epsilon_0 A}{2} E^2$$

- Energy density

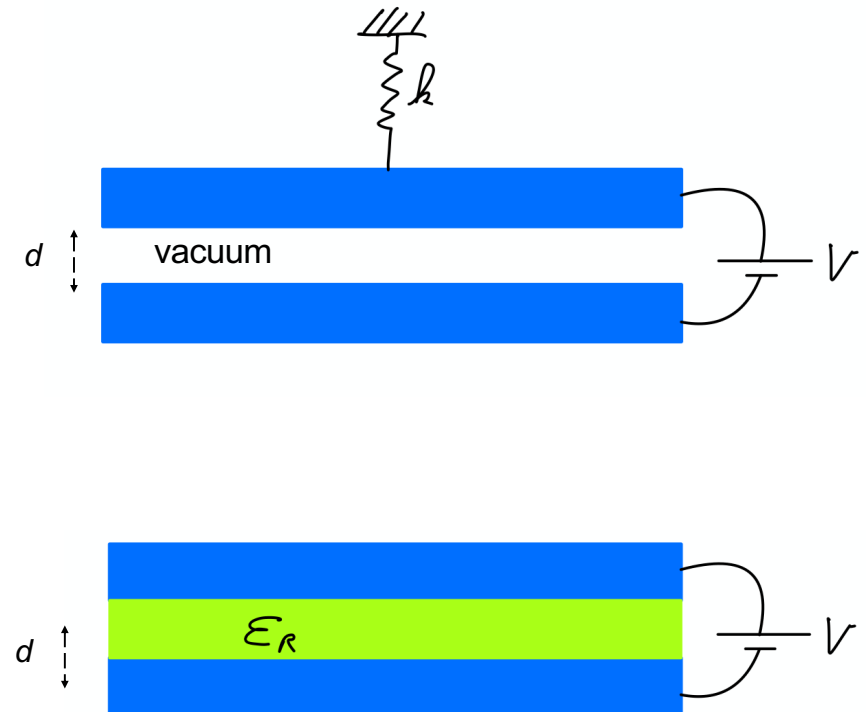
$$w_{ES} = \frac{1}{2} \epsilon_0 E^2$$

- If add a dielectric

$$w_{ES} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$$

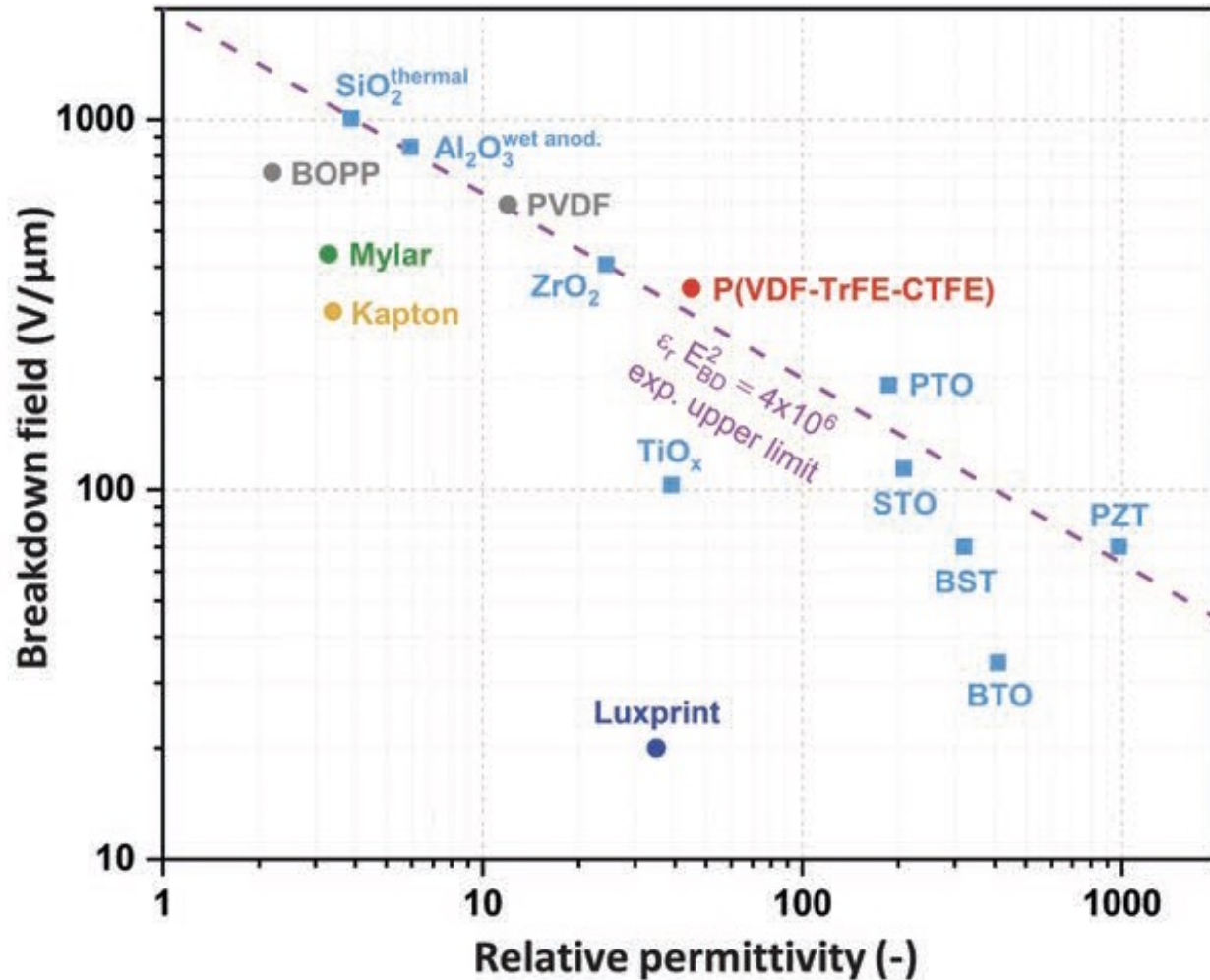
$$F_{ES} = \frac{1}{2} \epsilon_0 \epsilon_r A E^2$$

- Maximum energy density is limited by breakdown field E_{BD}



E =electric field, V =voltage,
 d =insulator thickness, A = electrode area

There is an empirical upper limit to $\epsilon_r \cdot E_{BD}^2$ product for solid dielectrics



- Want materials with high ϵ_r and high $E_{\text{breakdown}}$
- in MEMS, air is generally the dielectric to allow for motion
- But can also have solid dielectrics

Electrostatic actuation: energy density in air vs. in a solid

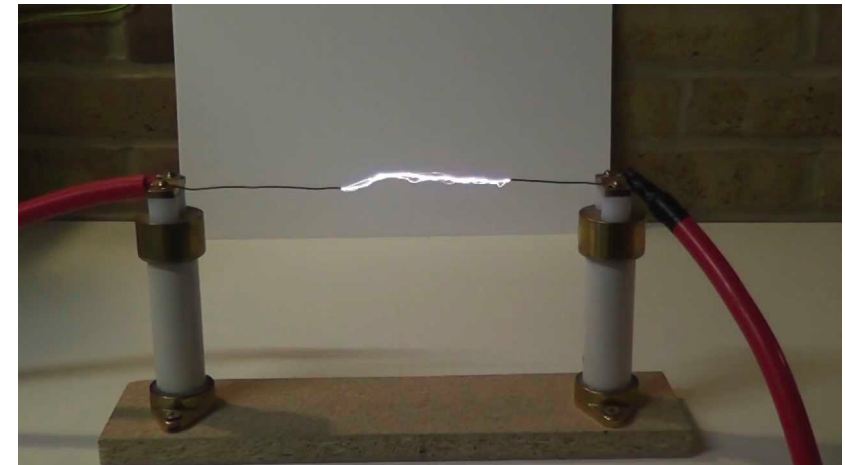
- Energy density $w_{es} = \frac{1}{2} \epsilon_0 E^2$ $w_{ES} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$

- Maximum energy density is limited by breakdown field

- in *air*, for large gaps, $E_{max} \approx 10^6 V/m$

$$w_{max} = \frac{1}{2} \epsilon_0 E_{max}^2 \cong 4.5 J / m^3 = 0.045 mbar$$

- But in **thin insulating films**, can have $E_{max} \approx 10^9 V/m$
i.e. $w_{max} = 45 bar$!

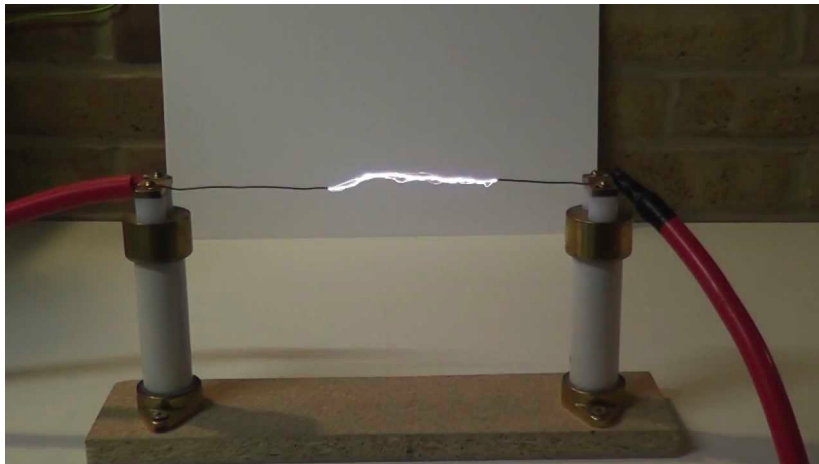


E : electric field
 E_{ES} : electrostatic energy
 w_{ES} : electrostatic energy density

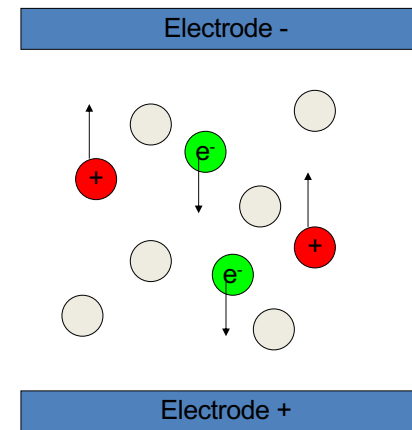
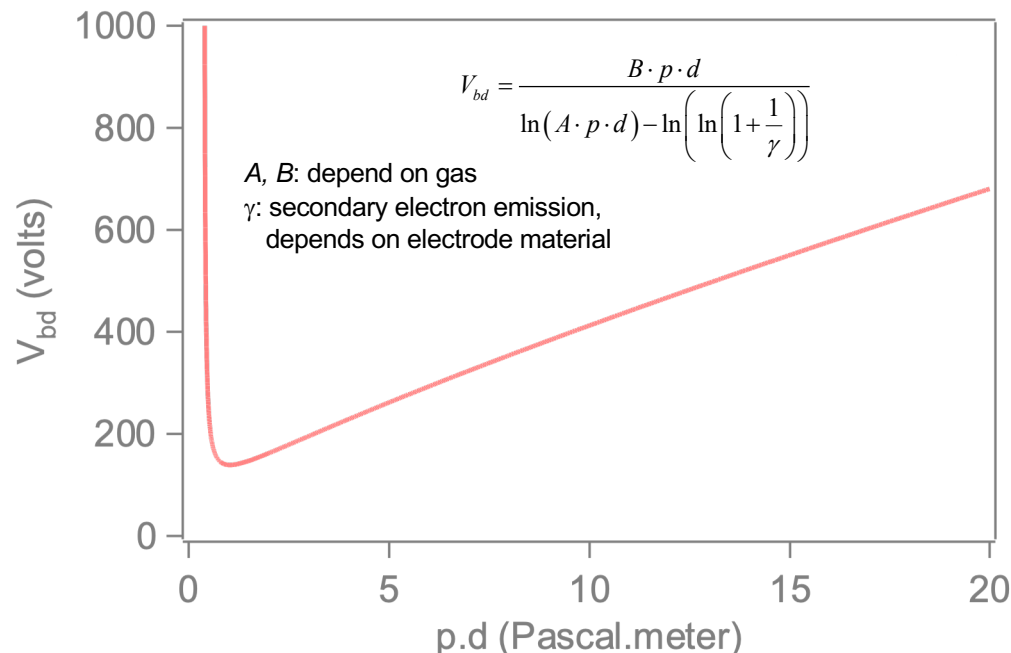
Air (>50 μm gap) : $3 \cdot 10^6 V/m$, polycrystalline Al_2O_3 : $2 \cdot 10^7 V/m$, thin film SiO_2 : $1 \cdot 10^9 V/m$

in MEMS/NEMS, plates are often separated by an air gap so they can move freely.

Why is the breakdown field lower in air than in solids?



Paschen Curve: $V_{\text{breakdown}}$ vs. Pressure.distance



Standard Paschen curve was derived for two large metal spheres, ignores field emission, fringing fields, etc.

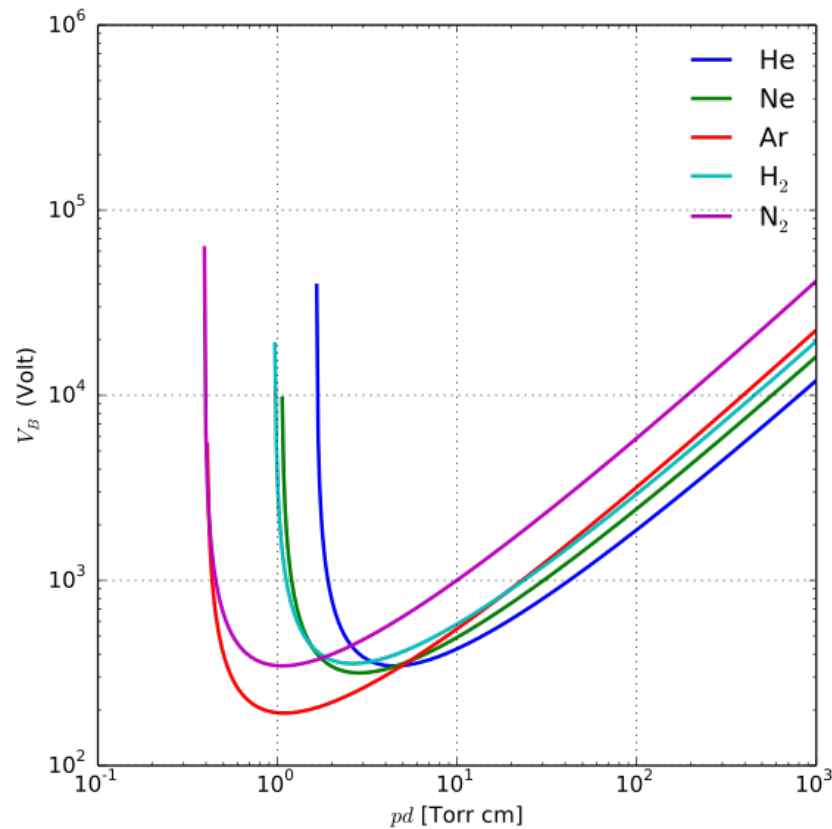
What is needed for ions to gain sufficient energy between collisions to ionize at impact and start avalanche?

Mean-free path is key

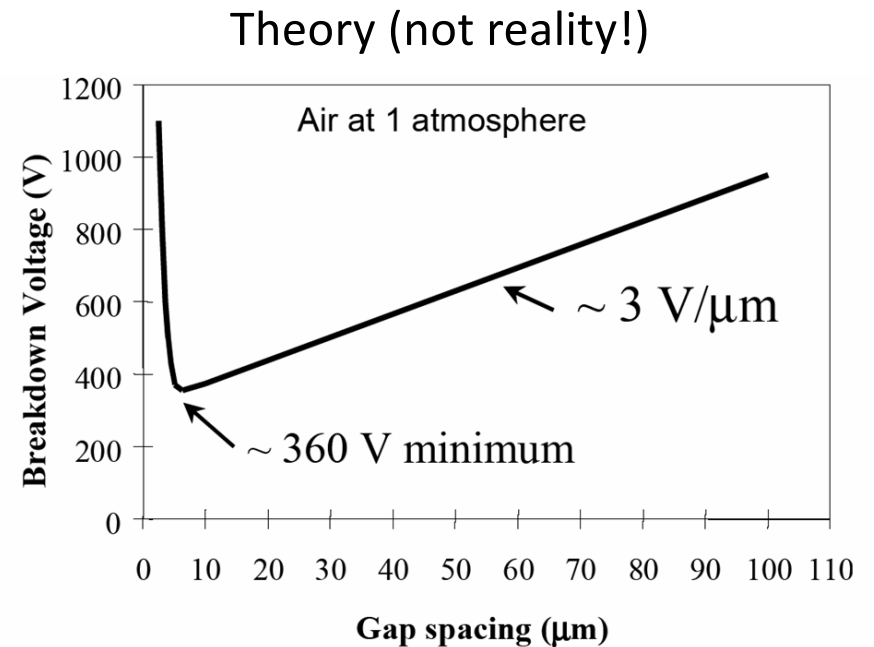
- At high P.d, there are sufficient gas molecules to allow for *Townsend avalanche breakdown* (ionization of gas by impact with electrons accelerated in electric field once they reach sufficient energy)
- At very low P.d, too few gas molecules to sustain avalanche: *vacuum isolation*

V_{bd} : breakdown voltage
 p : gas pressure
 d : gap between plates
 A, B : gas-dependent constants

What is the maximum voltage we can apply in air at 1 atm?

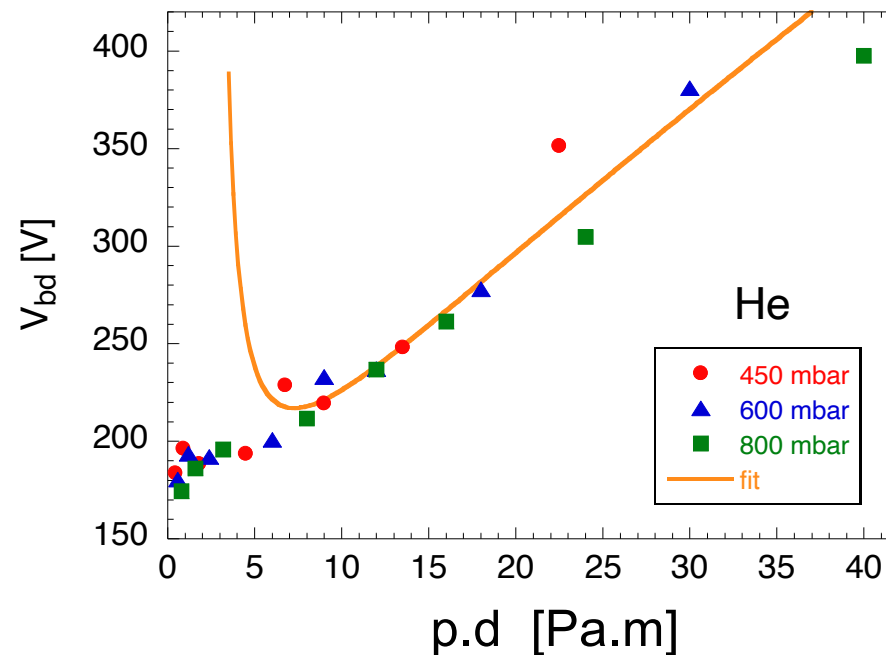


https://en.wikipedia.org/wiki/Paschen%27s_law

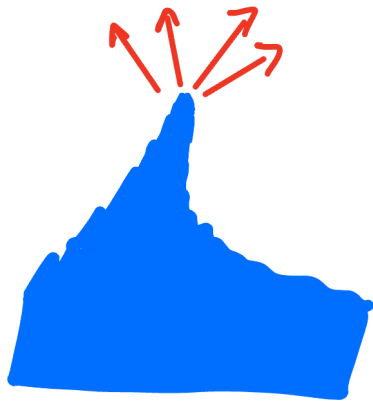
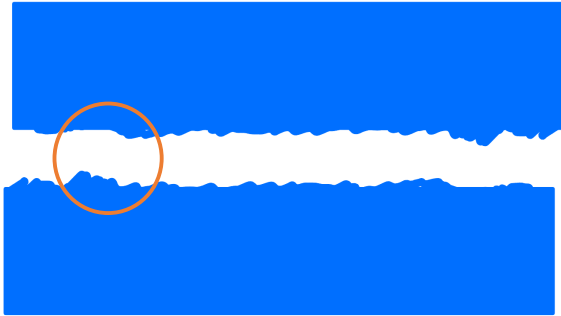


So then we never have breakdown if we operate below 360 V at 1 atm?

Measured Breakdown voltage for micromachined Aluminum electrodes (10 to 200 μm gaps)



(Carazzetti et al, SPIE Photonics West 2008)



- At scales of a few μm , we observe a breakdown V that decreases with decreasing gap
- This is mostly due to field emission of electrons
- 1 μm gap, 100 V: $E=10^8$ V/m
- But at surface asperities, get E field concentration, and $E \gg 10^9$ V/m
- A possible breakdown sequence:
 - i. Electrons emitted
 - ii. Joule heating due to current
 - iii. Atoms evaporate due to heat
 - iv. Now the vacuum is full of atoms...
 - v. ... avalanche breakdown

Breakdown V in air at 1 atm

important

in air at 1 atm:

- At large distances, E-field for breakdown is constant at $3 \times 10^6 \text{ V/m}$
- Paschen theory indicates a steep increase of V_{max} at sub $10 \mu\text{m}$ gaps,
- But in small gaps (few μm), V_{max} is not observed to increase again.

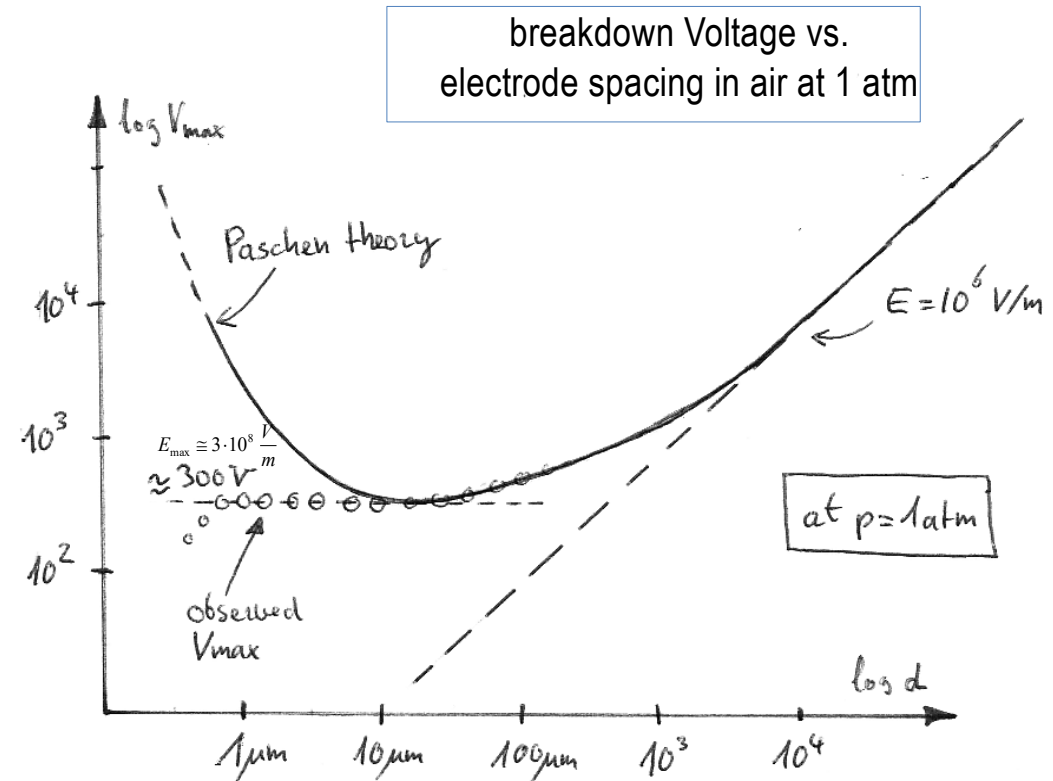
Electrode spacing at minimum breakdown voltage (@ 1 bar):

$h_{\text{min}} \cong 2\text{-}8 \mu\text{m}$ and voltage at h_{min} :

$$V_{\text{min}} \cong 300\text{V}$$

For 1-5 μm gap, the breakdown voltage is around 300V and the maximum electrical field is typically:

$$E_{\text{max}} \cong 3 \cdot 10^8 \frac{\text{V}}{\text{m}}$$



Paschen curve leads to very high energy density!

- For 1 to 2 μm gap, maximum electrostatic energy density is

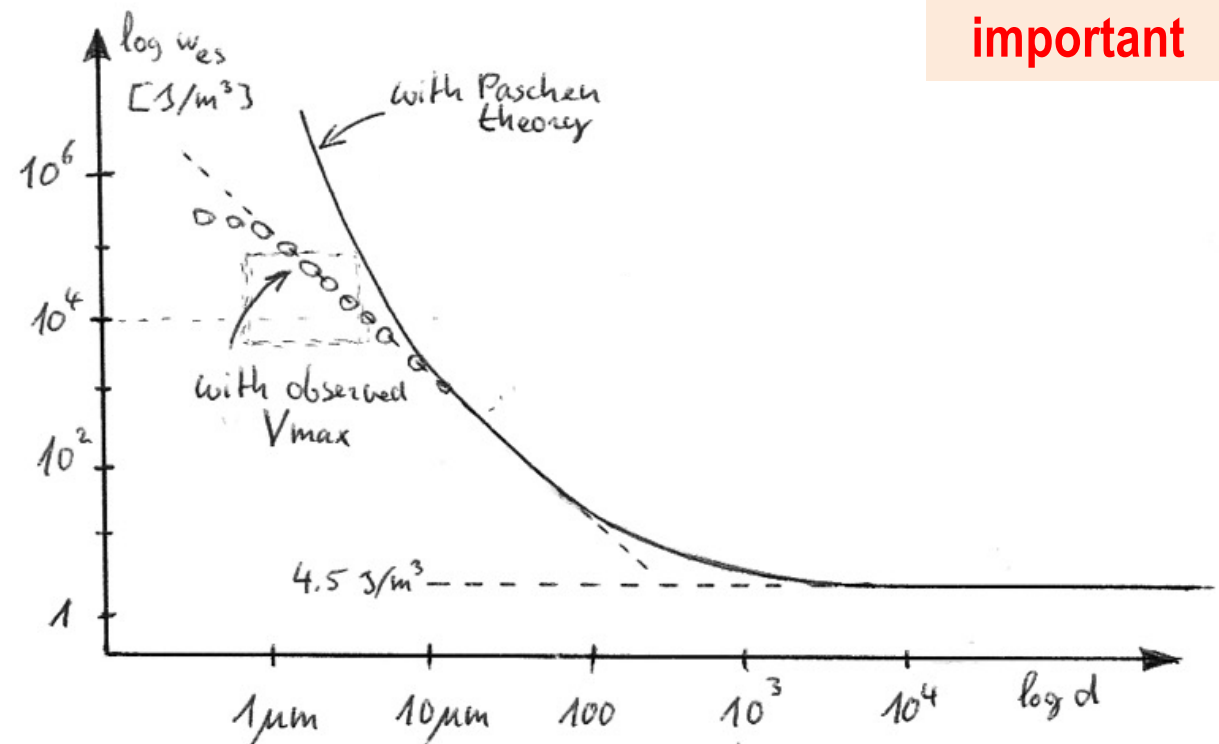
$$w_{\text{max}} = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 \cong 10^4 \text{ to } 10^5 \text{ J/m}^3$$

0.1 to 1 bar

- Compared with magnetic actuators, ES energy density is higher for sub-10 μm gaps

Can get very high actuation pressures thanks to high E field in small air gaps

important

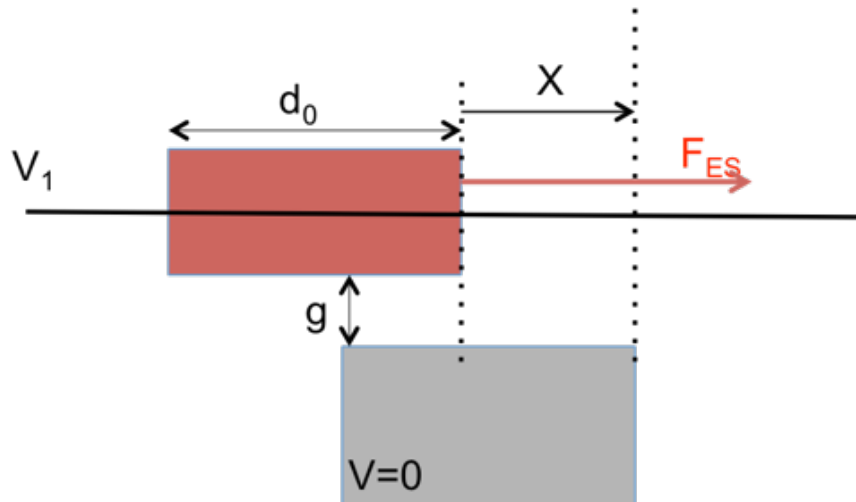


(rough) Comparison of energy densities between different actuation principles

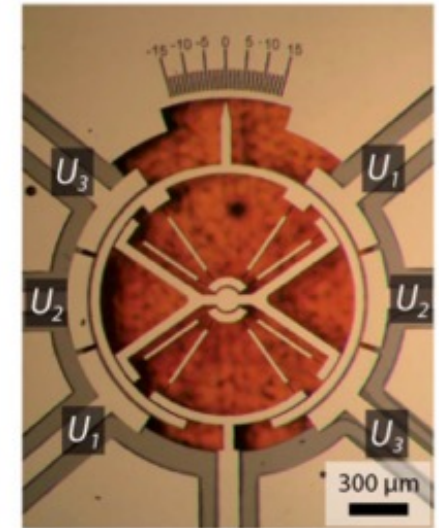
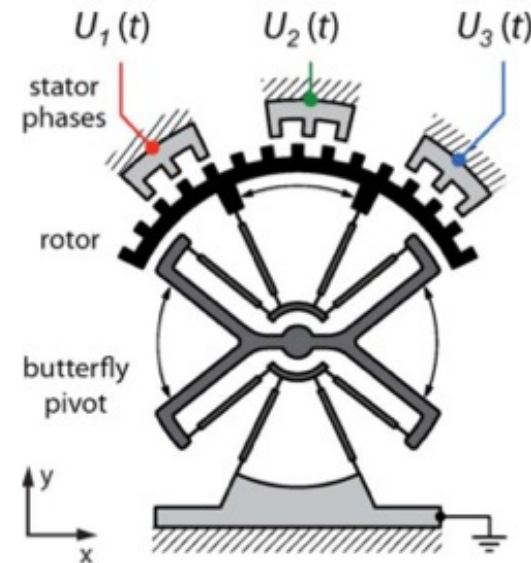
■ Electrostatics	(for small gaps)	$w_{\max} = \frac{1}{2} \varepsilon_0 E_{\max}^2 \cong 10^4 \text{ to } 10^5 \text{ J/m}^3$	
	(for large gaps)	$w_{\max} \cong 10^1 \text{ J/m}^3$	
■ Magnetic	(B_{sat} at 1T)	$w_{\max} = \frac{1}{2\mu_0} B_{\max}^2 \cong 10^6 \text{ J/m}^3$	(for size > cm)
■ Thermal Si	($\Delta T = 100^\circ \text{ C}$)	$w_{\max} = \frac{1}{2} Y (\alpha \cdot \Delta T)^2 \cong 5 \cdot 10^3 \text{ J/m}^3$	
■ Piezoelectric	($E_{\max} = 30 \text{ V}/\mu\text{m}$)	$w_{\max} = \frac{1}{2} Y (d_{33} \cdot E_{\max})^2 \cong 2 \cdot 10^2 \text{ J/m}^3$	
■ Pneumatic	($p_{\max} = 1000 \text{ bar}$)	$w_{\max} = p_{\max} \cong 10^8 \text{ J/m}^3$	
■ Mammalian Muscle		$w_{\max} = p_{\max} \cong 10^6 \text{ J/m}^3$	

Examples of ES actuator with constant gap (but varying overlap)

Electrostatic devices with fixed gap spacing (only change electrode overlap, not gap)



- red block at voltage V_1 can slide on a rail, at fixed gap g from grounded, fixed gray block,
- ES force “lines up” the two blocks.
- If have three offset positions, can make a stepper motor.



$$C = \epsilon\epsilon_0 \frac{(d_0 - x)t}{g}$$

$$F = -\frac{dE}{dx} = -\frac{d}{dx} \left(\frac{1}{2} CV^2 \right) = \frac{1}{2} \epsilon\epsilon_0 V^2 \frac{t}{g}$$

Scaling: for constant V , $F \sim L^0$, for constant E , then $F \sim L^2$
(but we can't reach the same E at large scale as for μm scale)

Stranczl, M.; Sarajlic, E.; Fujita, H.; Gijss, M.A.M.; Yamahata, C.,
"High-Angular-Range Electrostatic Rotary Stepper Micromotors Fabricated With SOI
Technology," JMEMS, vol.21,, pp.605 2012 doi: 10.1109/JMEMS.2012.2189367

Electrostatic 3-phase Linear Stepper Motor Fabricated by Vertical Trench Isolation Technology

Edin Sarajlic, Christophe Yamahata,
Mauricio Cordero and Hiroyuki Fujita

© 2009/02, the University of Tokyo

Stranczl, M.; Sarajlic, E.; Fujita, H.; Gijs, M.A.M.; Yamahata, C., JMEMS , vol.21, no.3, pp.605,620, 2012 doi: 10.1109/JMEMS.2012.2189367

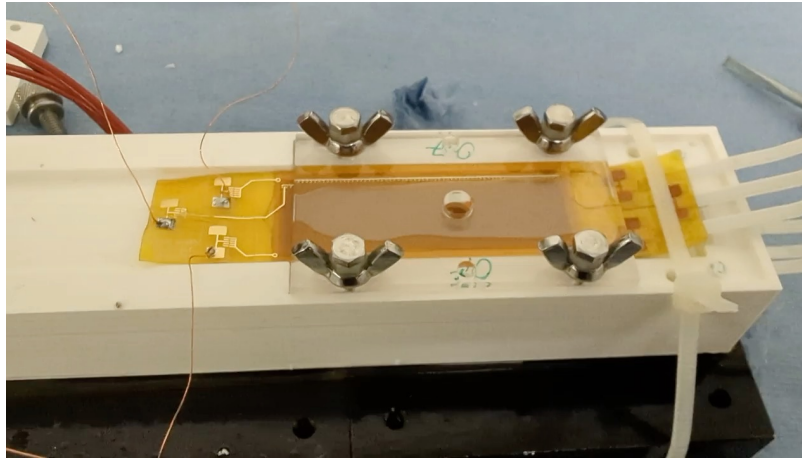
MEMS in air

Macro-scale linear electrostatic motors

Electrode Gaps: 40-80 μm for high energy density

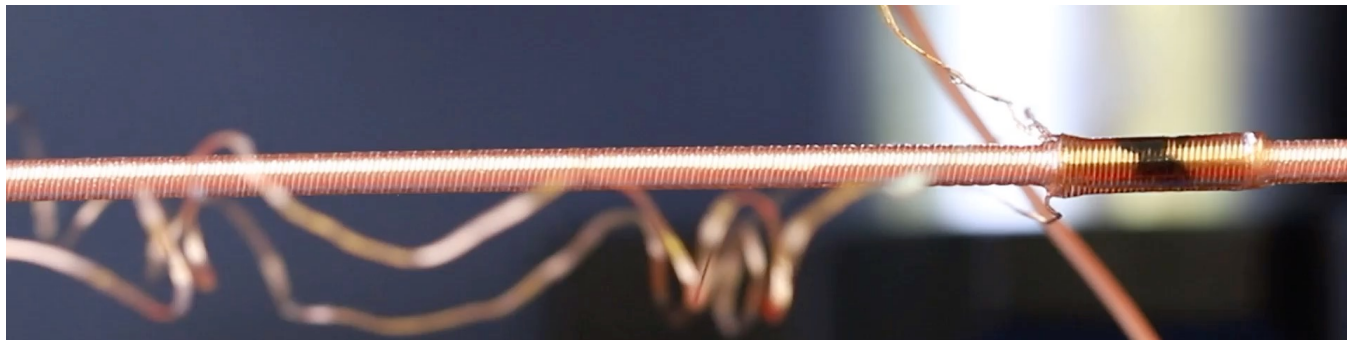
Immersed in oil for higher breakdown field

Ribbon Format



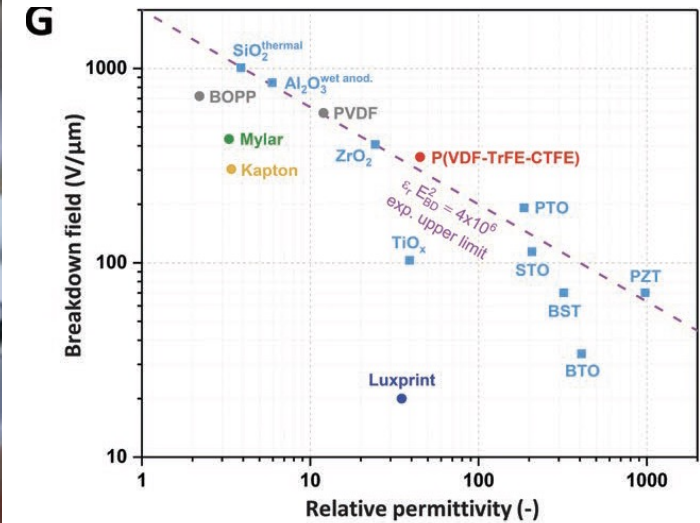
M. Schouten

Fiber Format



S. Schaller

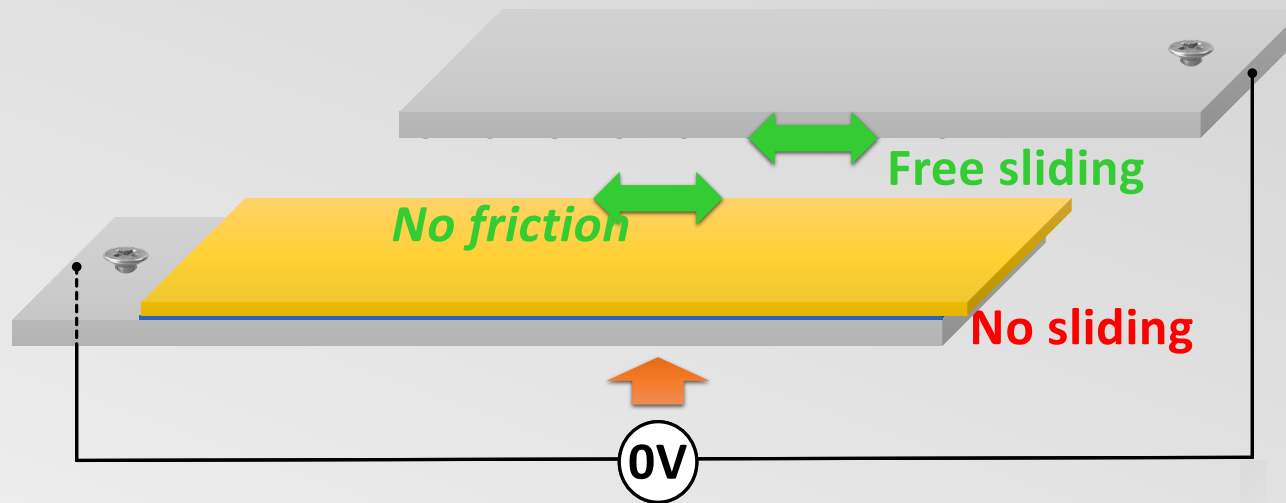
High-permittivity dielectrics for high ES forces



Haptic glove for VR + AR:
The clutch actively blocks finger motion to make virtual objects feel solid.

Hinchet and Shea, Adv. Mat. Tech. 2019
Hinchet and Shea, Adv. Intell. Sys. 2022

How does the ES clutch block motion?



No voltage

➤ Finger is free

Voltage on

➤ Finger is blocked

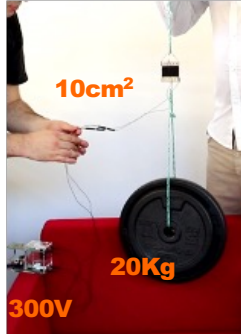
$$F_{F\max} = \mu \times F_{N\max} = \frac{\epsilon_0 A}{2} \times \mu \epsilon_r E_{bd}^2$$

Textile ESclutch can block 2 kg/cm² at 300 V

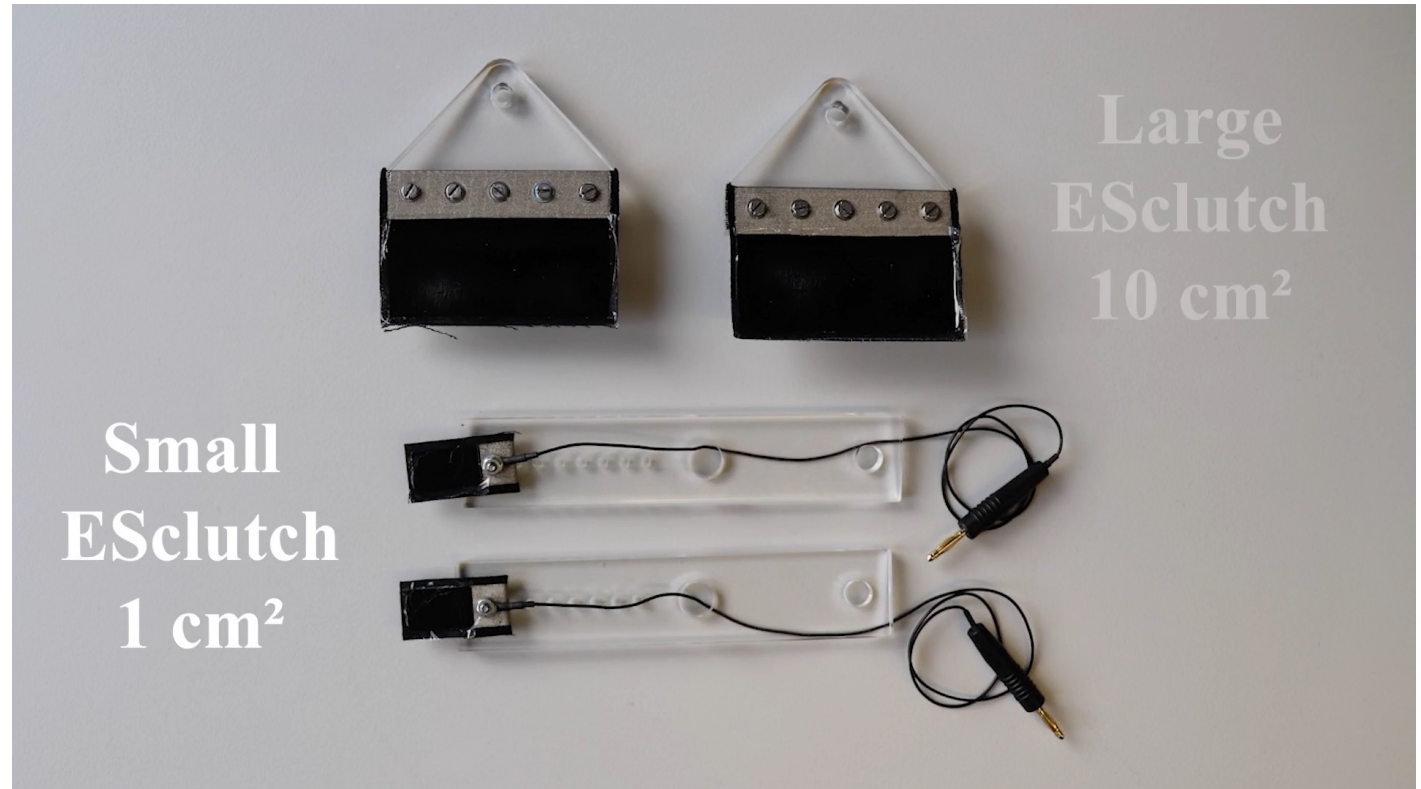
Small ESclutch



Large ESclutch



- High holding force :
20 N/cm² at 300 V
- low power **1.2 mW/cm²**
- Flexible, Lightweight **30 mg/cm²**
- Fast **< 15 ms**



- Performance comes from use of $\epsilon_r=40$ material, with $E_{BD} > 100$ V/ μ m
- mW power enables use in in exoskeletons and full-body haptics
- Textile format

cm scale device, but 10 μ m gap between electrodes...

Hinchet and Shea, Adv. Mat. Tech. 2019

Parallel plates electrostatic actuator. Gap changes

(electrodes move in direction of E field, no change in overlap)

1. Equilibrium position
2. Effective spring constant
3. Pull-in phenomena

Static equilibrium position for small displacements

$$F_{\text{elast}} = F_{\text{electrostatic}}$$

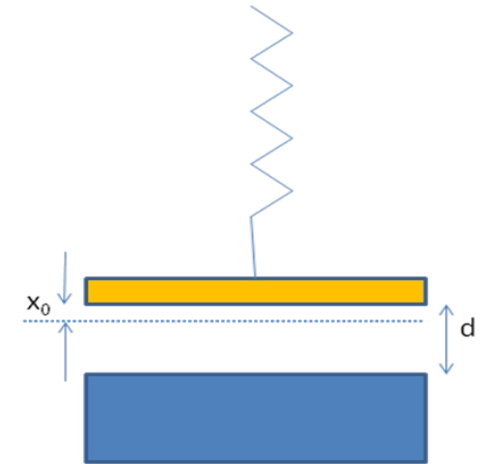
$$k \cdot x_0 = \frac{\epsilon_0 A V^2}{2(d - x_0)^2}$$

Spring
force

Electrostatic
force

$$x_0 = \frac{\epsilon_0 A V^2}{2k d^2} = \frac{Q^2}{2\epsilon A k}$$

(only valid if $x \ll d$)



(Q= charge)

Influence of electric field on **effective** spring constant

$$F(x) = -k_0 x + \frac{\epsilon_0 A V^2}{2(d-x)^2}$$

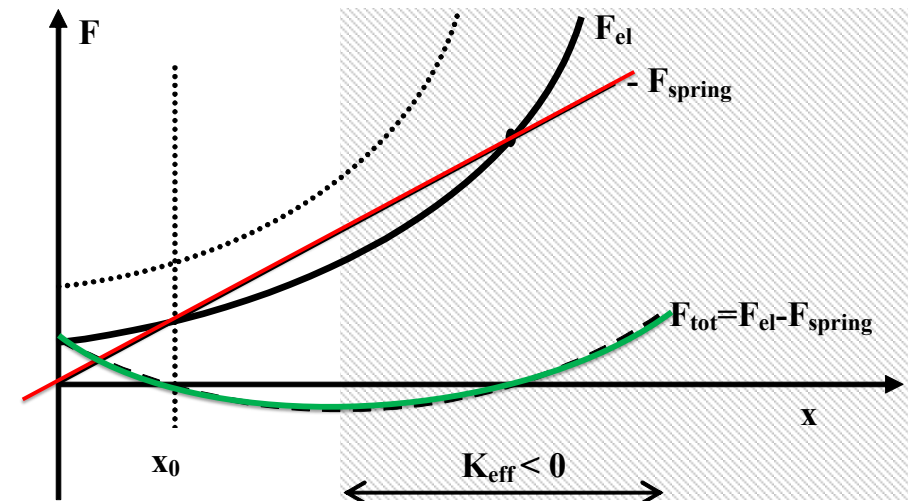
- With non-linear electrostatic force, the effective system spring stiffness decreases
- Effective spring constant of system:

$$k_{eff} = -\frac{dF}{dx} = k - \frac{\epsilon_0 A V^2}{(d-x)^3}$$

- With a bias voltage V , there is an apparent spring “softening” and thus a decrease of the natural frequency of oscillators:

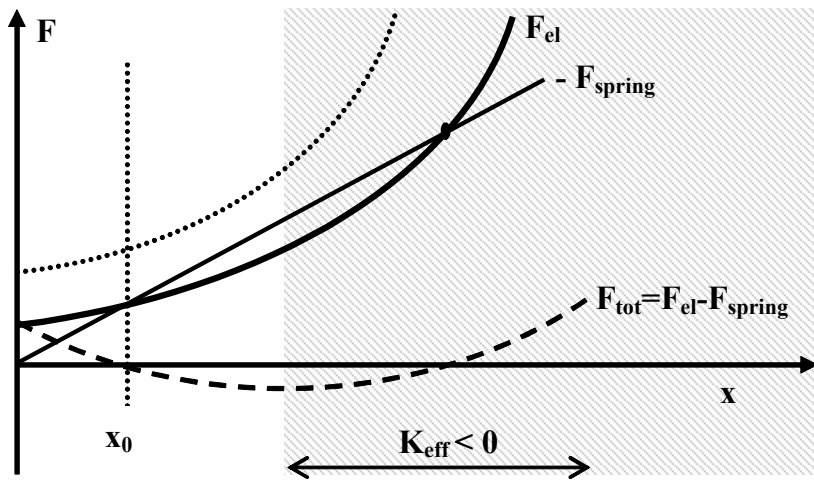
$$\omega_{res}(V) = \sqrt{\frac{k_{eff}}{M}} = \sqrt{\frac{1}{M} \left(k - \frac{\epsilon_0 A V^2}{(d-x)^3} \right)} \quad \text{for small displacements} \quad k_{eff} = k - \frac{k \cdot x_0}{d} = k \left(1 - \frac{x_0}{d} \right)$$

- (good news) we can tune ω_0 by applying a bias voltage
- (bad news) we have an undesired shift in ω_0 at large displacements...

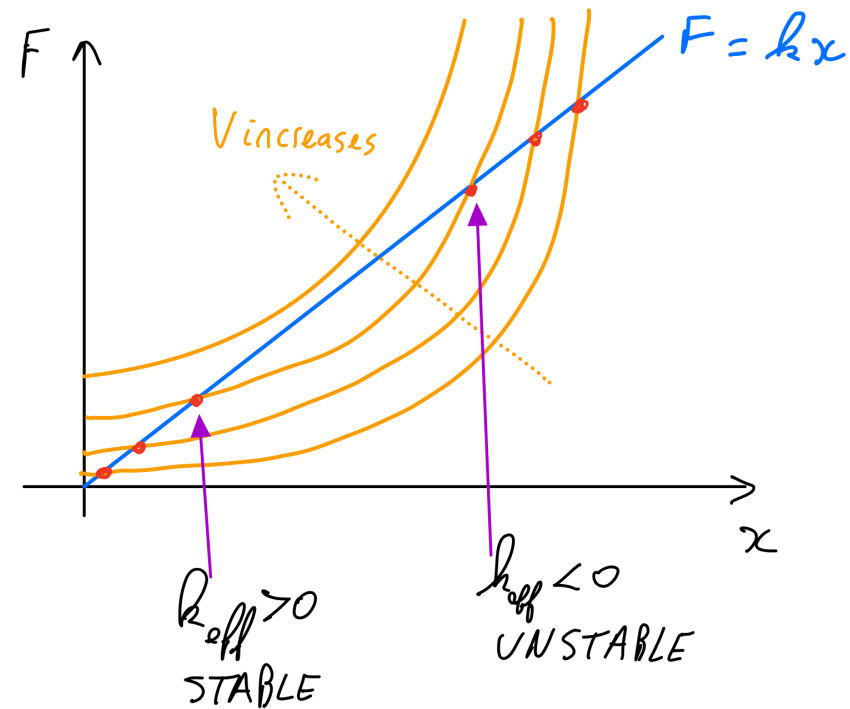


(figure from S. Senturia's book)

Pull-in voltage 1/2



(figure from S. Senturia's book)



$$F_{tot} = kx_0 + \frac{\epsilon_0 AV^2}{2(d-x_0)^2} = 0$$

$$kx_c = \frac{\epsilon_0 AV^2}{2(d-x_c)^2}$$

2 equilibrium positions
(but only 1 stable)

$$k_{eff} = -\frac{dF_{tot}}{dx} = k - \frac{\epsilon_0 AV^2}{(d-x)^3}$$

Condition for stability: $k_{eff} > 0$

Pull-in voltage 2/2

Maximal stable position x_c when $k_{eff}(x_c) = 0$

$$k = \frac{\varepsilon_0 A V^2}{(d - x_c)^3} = \frac{\varepsilon_0 A V^2}{2(d - x_c)^2} \frac{2}{(d - x_c)} = \frac{2kx_c}{(d - x_c)}$$

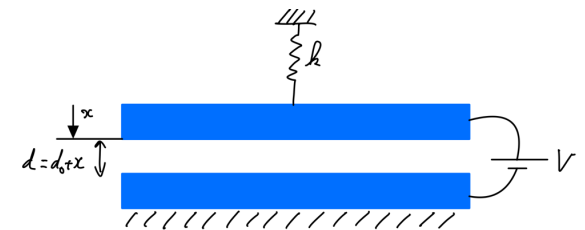
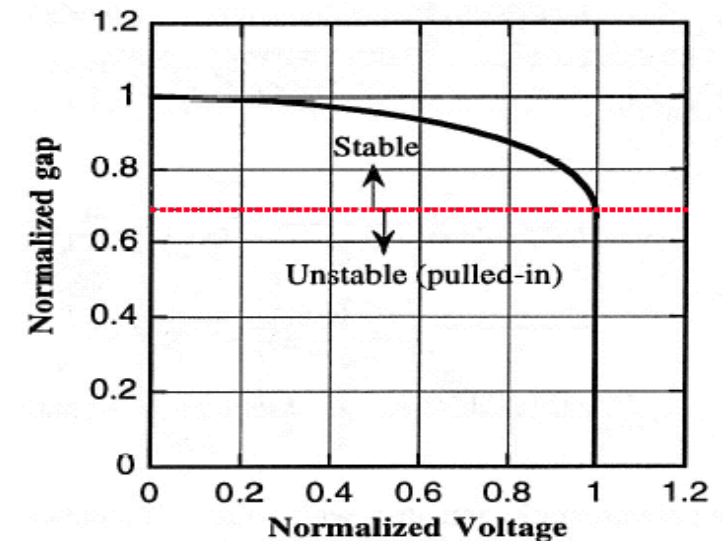
$$x_c = \frac{1}{3}d$$

$$V_{pull} = \sqrt{\frac{k}{\varepsilon_0 A} \left(\frac{2}{3}d\right)^3}$$

$$V_{pull} \propto d^{3/2}$$

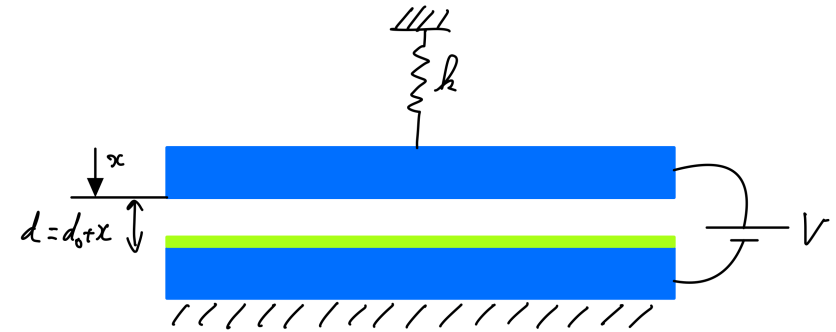
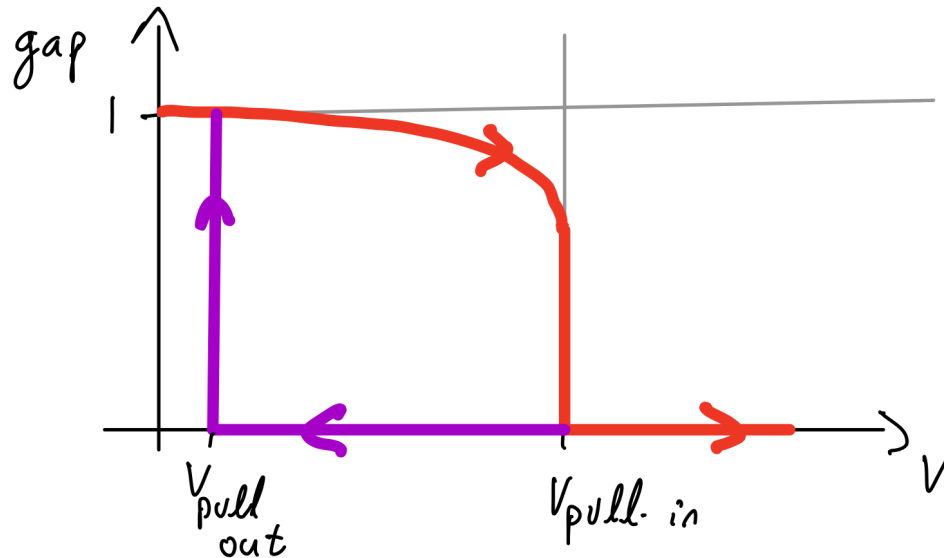
$$V_{pull} \propto L$$

- **Only 1/3 of the gap can be used for actuation!**
- For doubly clamped beam, the pull-in limit can be up to $\frac{1}{2}h$ because of non-linearity of mechanical restoring force.
- Stoppers or a dielectric film are needed to prevent short circuit when snapping in



$V_{pull-in}$ is a key design parameter for Electrostatic MEMS (spacing, size, shape...)

Pull in can be a desired feature (or a nuisance)

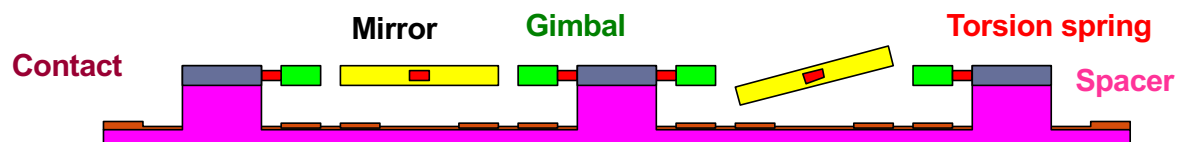
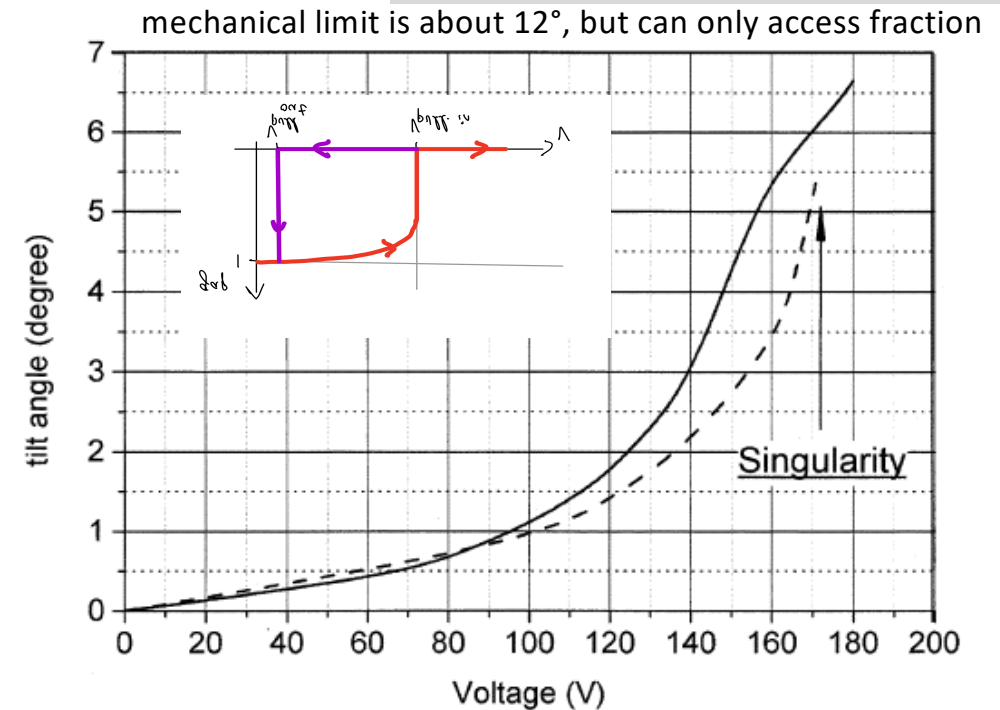
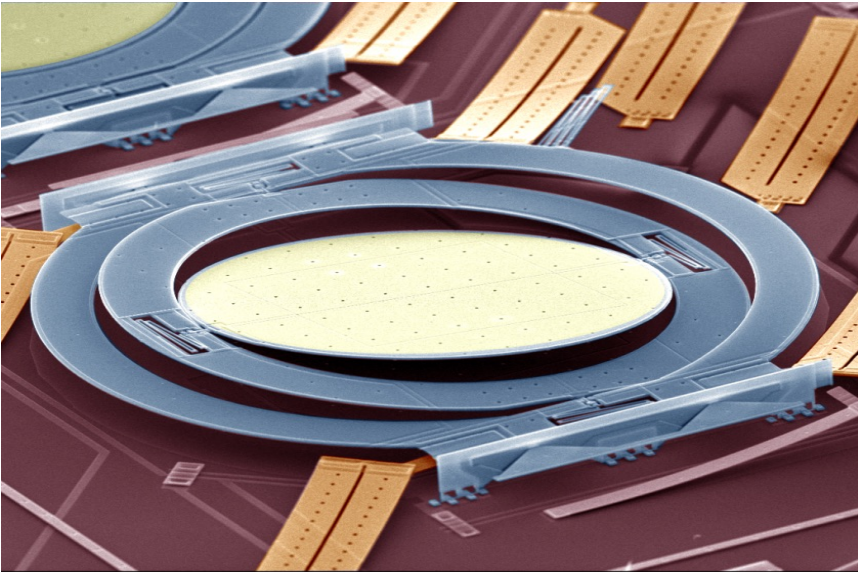


Hysteresis for pull-out (if have thin dielectric layer at bottom of gap to prevent a short circuit)

$$V_{pull-in} = \frac{2}{3} \sqrt{\frac{2kd^3}{3\epsilon_0 A}}$$

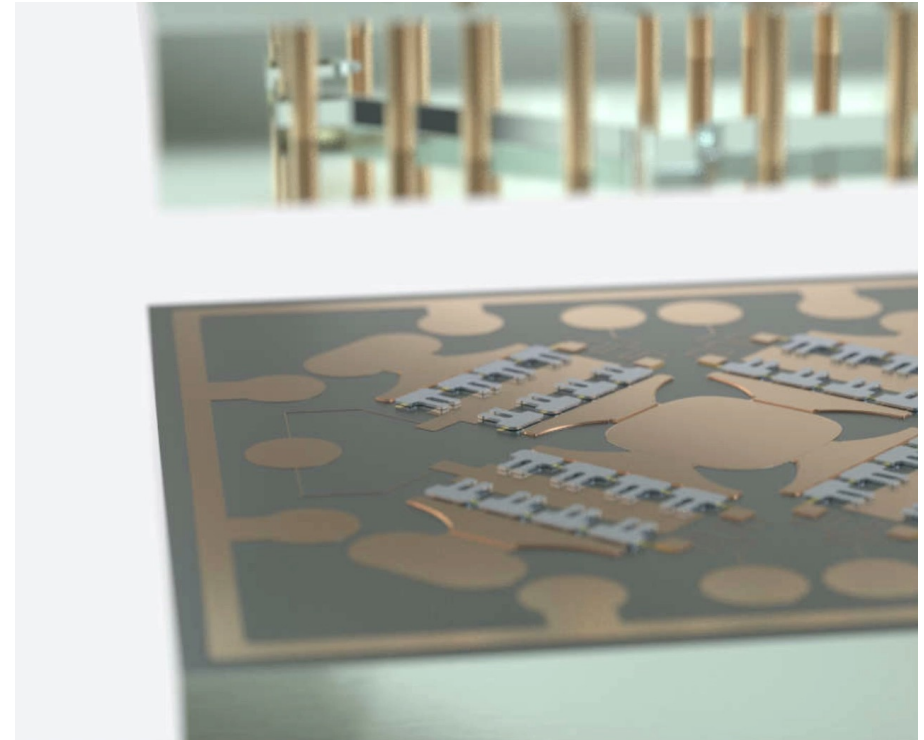
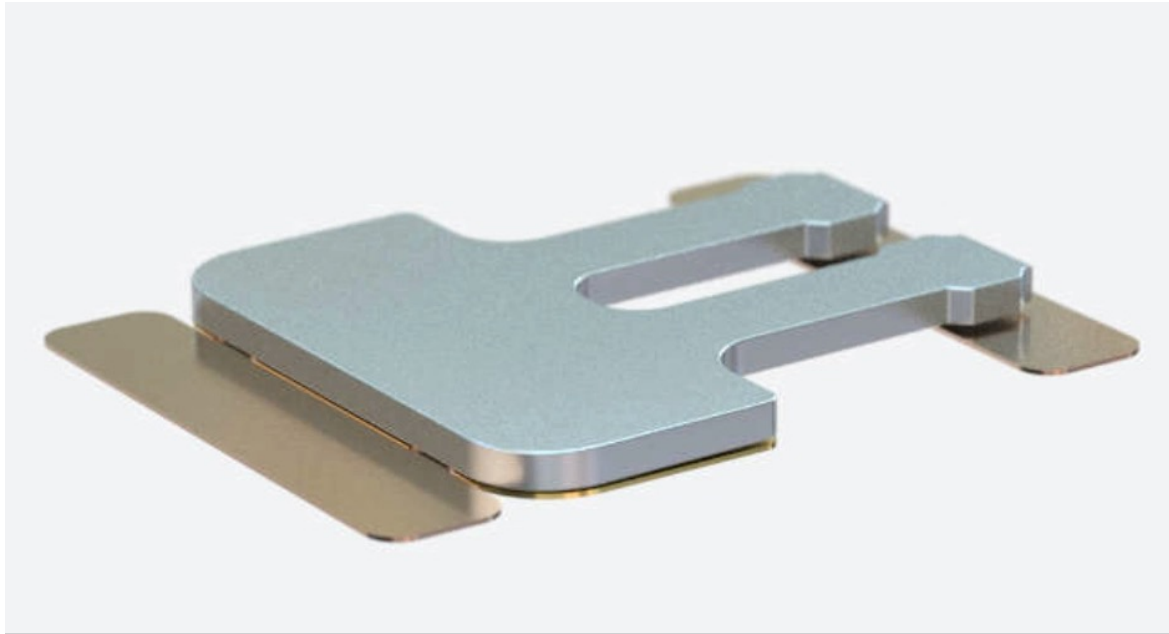
$$V_{pull-out} \sim t_{dielectric} \sqrt{\frac{2kd}{\epsilon_{dielectric} A}}$$

Here pull-in is not desired, as it limits range over which angle can be controlled)



V. Aksyuk et al. (2003). Beam-steering micromirrors for large optical cross-connects. *Journal of Lightwave Technology*, 21(3), 634–642.

Electrostatic micro-relay (pull-in is a feature)



Carries AC/DC and RF



Linear performance from DC to > 50 GHz



Switches at <10 μ s



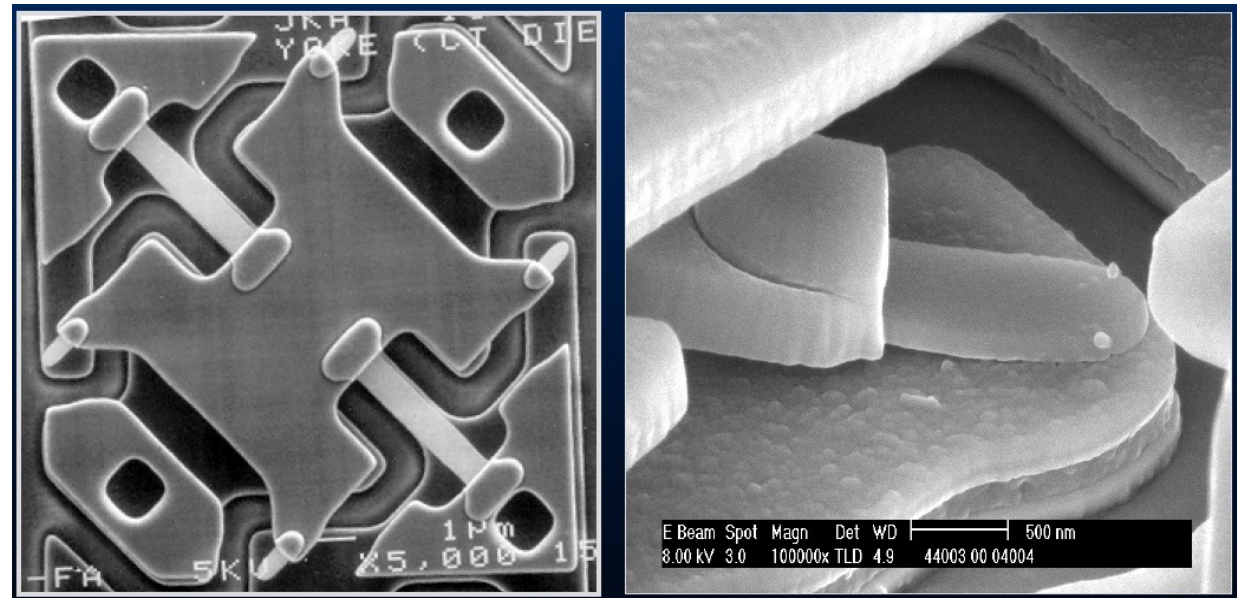
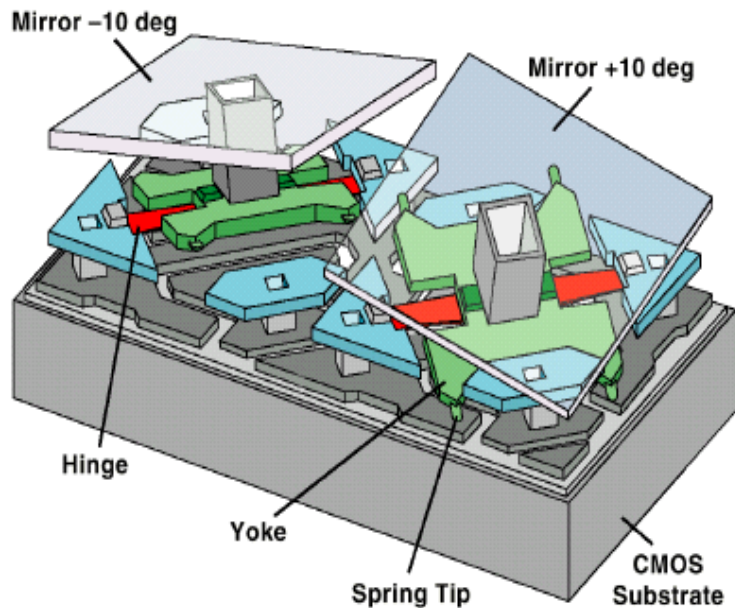
Performs reliably >3 Billion times



Negligible resistance eliminates the need for bulky, heavy heatsinks



TI's Digital MicroMirror devices: DMD. Relies on pull-in: does not need a precise voltage to get precise angle

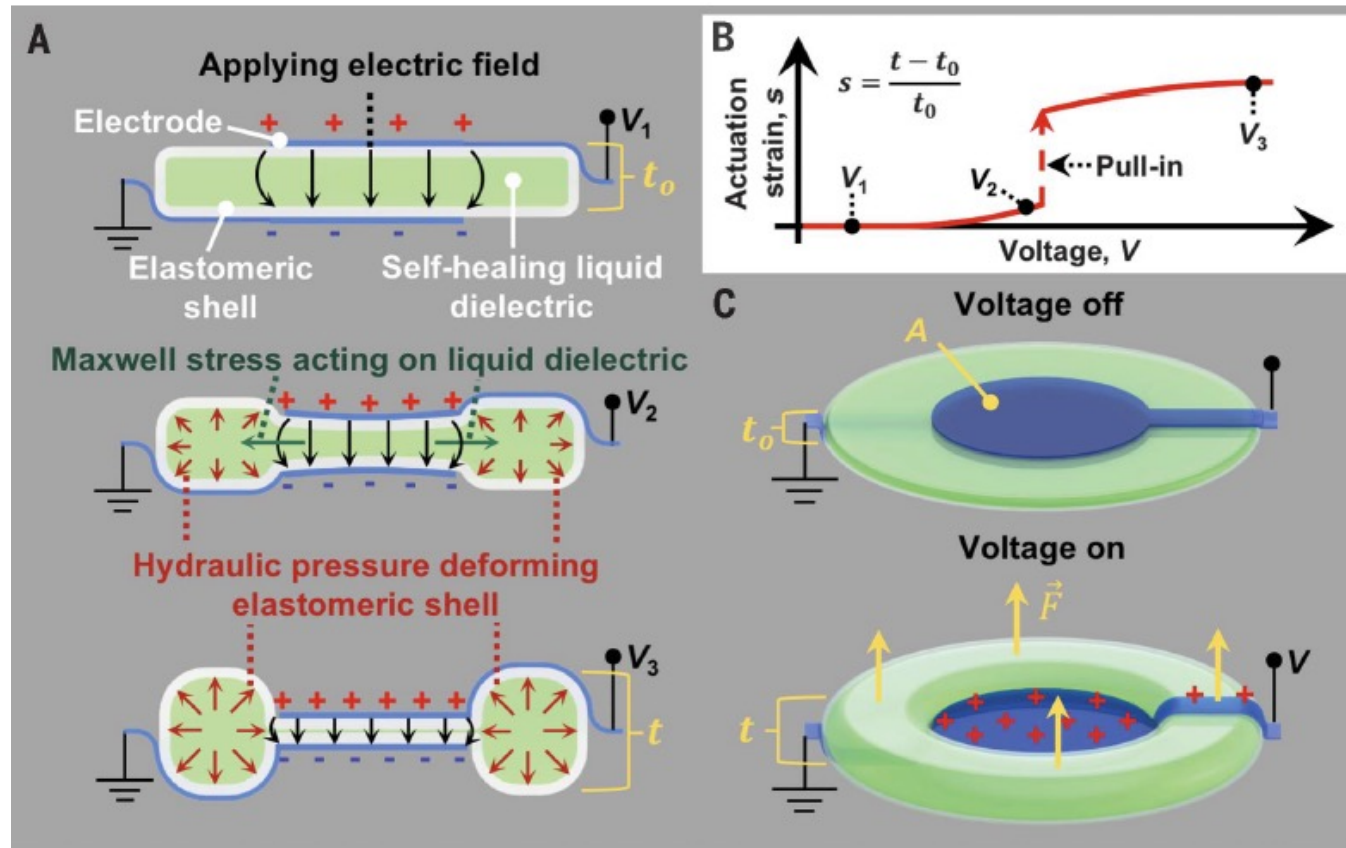


(pixel size approx. $10\ \mu\text{m} \times 10\ \mu\text{m}$)

- Underlying process is: CMOS + CMP planarization + Al-alloy mirror
- The hinge is only 60-100 nm thick (only 2-3 grains thick => no fatigue)
- Resonance frequency 50-200 kHz
- Anti-stiction PFDA self assembled monolayer
- **Gap of a few μm : can operate at 15-20 V, near max energy density from Paschen curve**

Need to address > 1 million pixels: need very high integration

Liquid dielectric: tolerant to breakdown, built-in hydraulic amplification

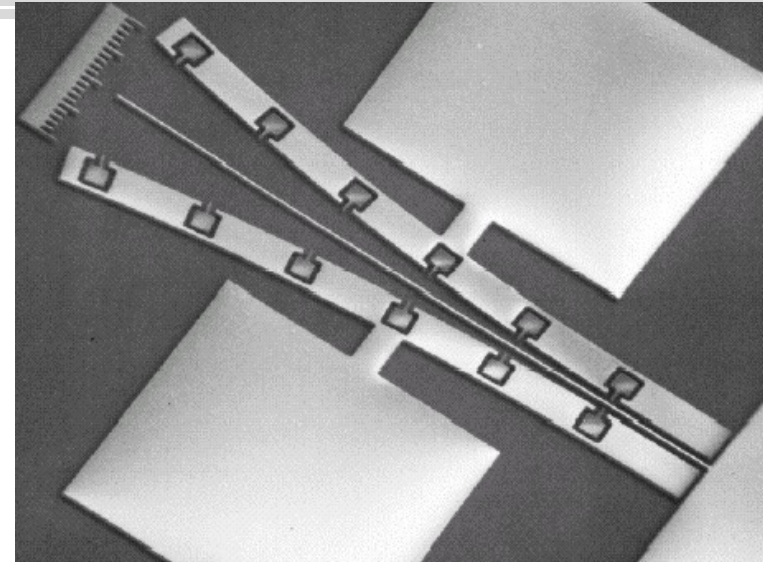
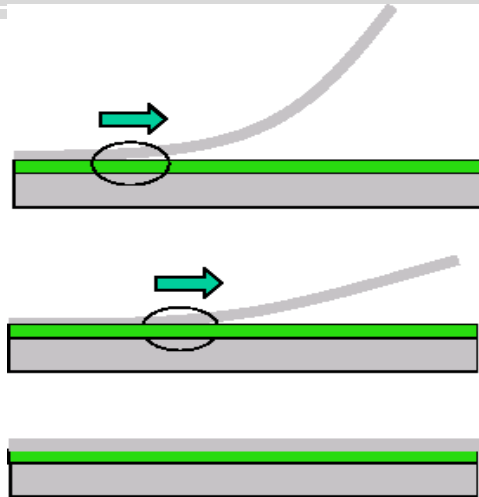


Acome, et al . “Hydraulically Amplified Self-Healing Electrostatic Actuators with Muscle-like Performance. ” *Science* 359, no. 6371 (2018): 61–65. <https://doi.org/10.1126/science.aao6139>.

Electrostatic Zipping actuators are one way to:

- reduce voltage of electrostatic actuators
- while keeping large displacement
- Use materials with higher ϵ_r and higher E_{BD}

Zippering actuators, silicon MEMS



- High ES force generation, since always have a small gap
 - Long-distance and stable displacement
 - Possibly lower voltage drive (since small gap)
 - Gain ES energy vs. Mechanical energy
- Limited use in MEMS due to stiction
- J. Branebjerg, P. Gravesen, "A New Electrostatic Actuator providing improved Stroke length and Force", MEMS'92, Travemünde (Germany)

Electrostatic zipping devices, macro-scale (but μm insulator)

Electro-ribbon actuators and electro-origami robots

Majid Taghavi, Tim Helps, Jonathan Rossiter

Movie S2 | Isotonic and isometric actuation of a standard electro-ribbon actuator.

(A), A standard electro-ribbon actuator lifts a 20 g mass 51.75 mm. Applied voltage is 8 kV. Contraction is 99.31 %.

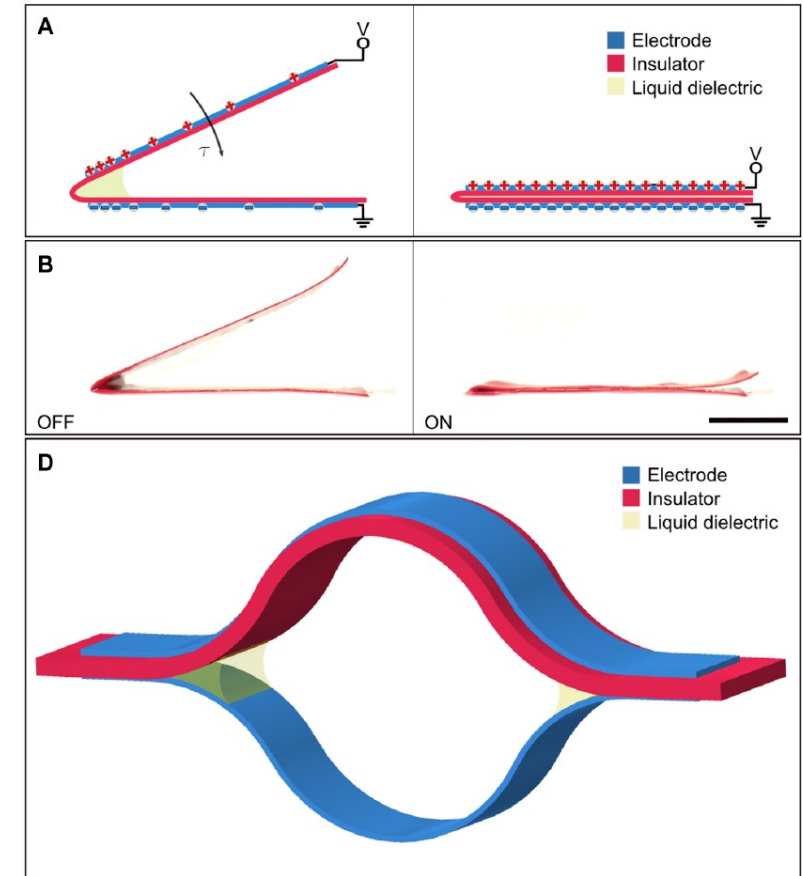
(B), Isometric testing of a standard electro-ribbon actuator. Applied voltage is a step input, starting at 1 kV and increasing by 1 kV every five seconds to a maximum voltage of 6 kV. The actuator extension is held constant at 24 mm.



University of
BRISTOL



Bristol Robotics Laboratory

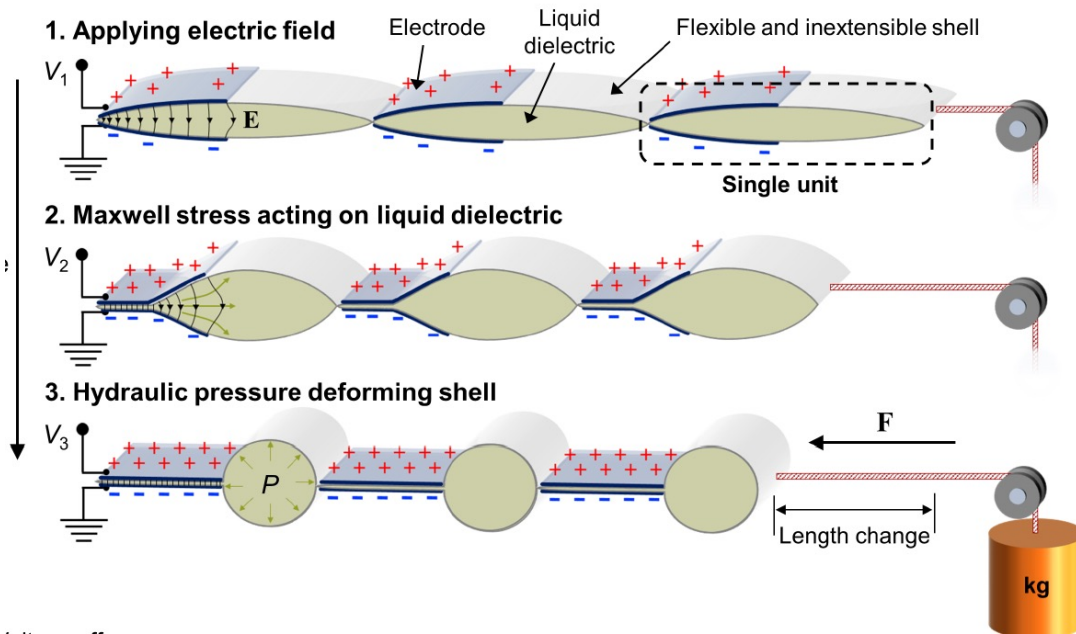


- Electro-ribbon actuators (Rossiter group)
- “Electro-origami principle [dielectrophoretic liquid zipping]”

M. Taghavi, T. Helps, J. Rossiter, Electro-ribbon actuators and electro-origami robots. *Science Robotics*. **3**, eaau9795 (2018).

Electrostatic zipping devices, macro-scale (but μm insulator)

- Peano-HASEL (Keplinger group)



Supplementary Movie 1

Actuation of a twelve-unit
HS-Peano-HASEL actuator

Keplinger Research Group

University of Colorado
Boulder

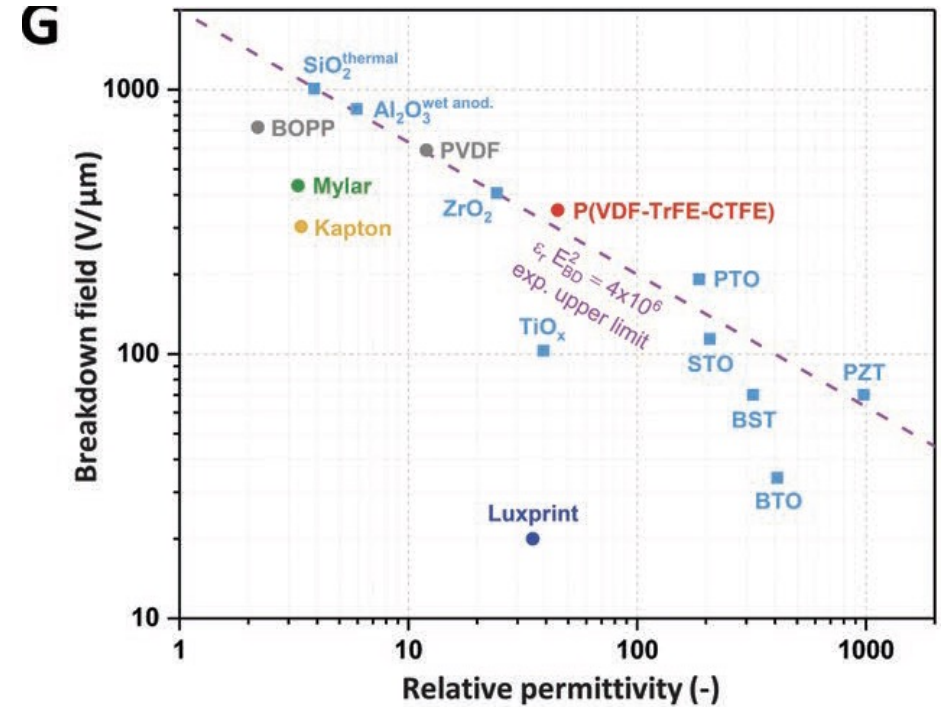
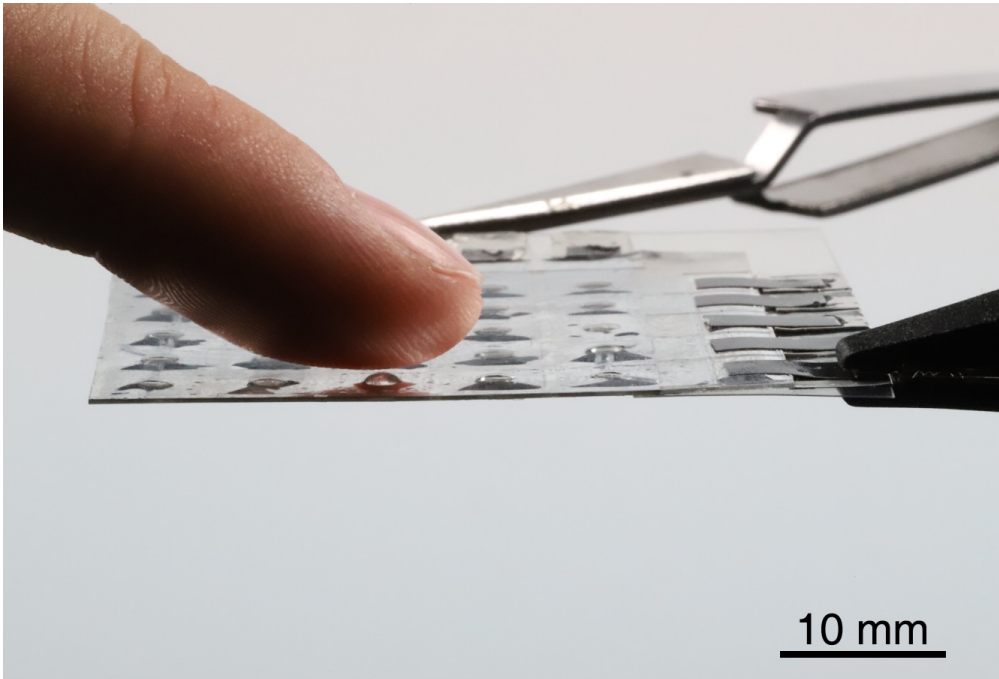
同济大学
TONGJI UNIVERSITY

In all these zipping devices, the dielectric layer is 10-50 μm thick. Generally thickness not scaled down/up as area changes.

N. Kellaris, V. G. Venkata, G. M. Smith, S. K. Mitchell, C. Keplinger, Peano-HASEL actuators: Muscle-mimetic, electrohydraulic transducers that linearly contract on activation. *Science Robotics*. **3**, eaar3276 (2018).

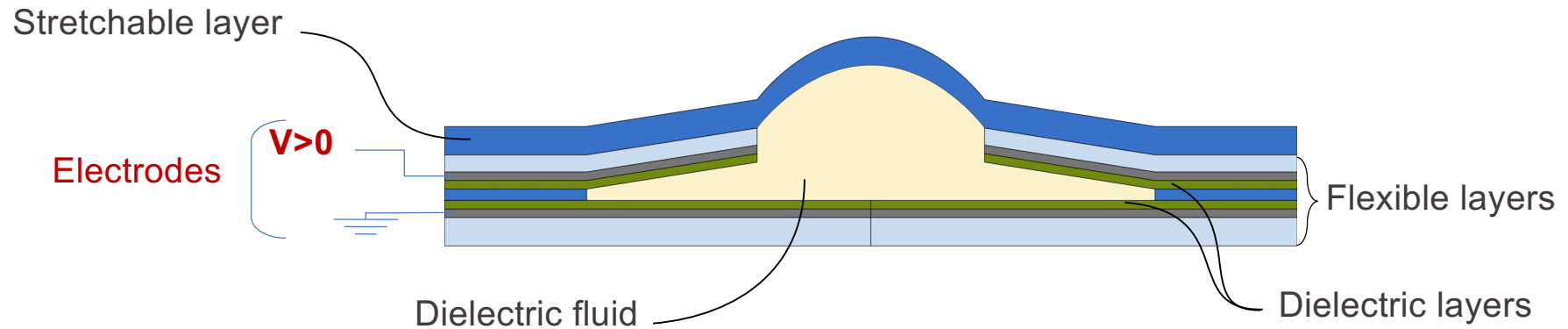
Electrostatic zipping devices

- HAXEL (EPFL-LMTS)



Leroy and Shea, Adv. Mat 2020
doi: 10.1002/adma.202002564

HAXELs: Hydraulically Amplified electrostatic taXELs



- Non-stretchable electrode and $\epsilon_r=40$ dielectric
- Built-in hydraulic amplification
- But central stretchable silicone region

E. Leroy et al, Adv. Mat 2020

E. Leroy et al, Adv. Mat Tech 2023

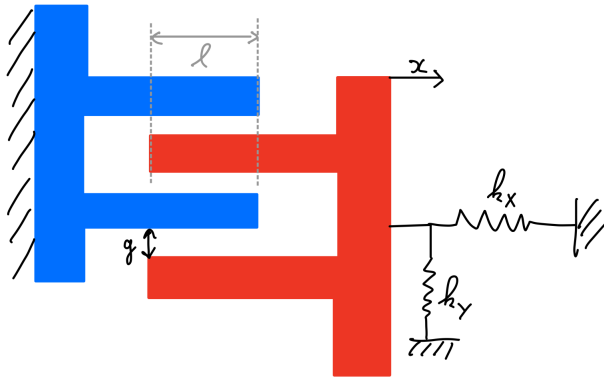
Hydraulically amplified mm-scale actuators for wearable haptics

Edouard Leroy and Herbert Shea

Ecole Polytechnique Fédérale de Lausanne
Neuchatel, Switzerland

COMB DRIVE ACTUATORS

Comb drive: overcomes many limitations of parallel plate MEMS

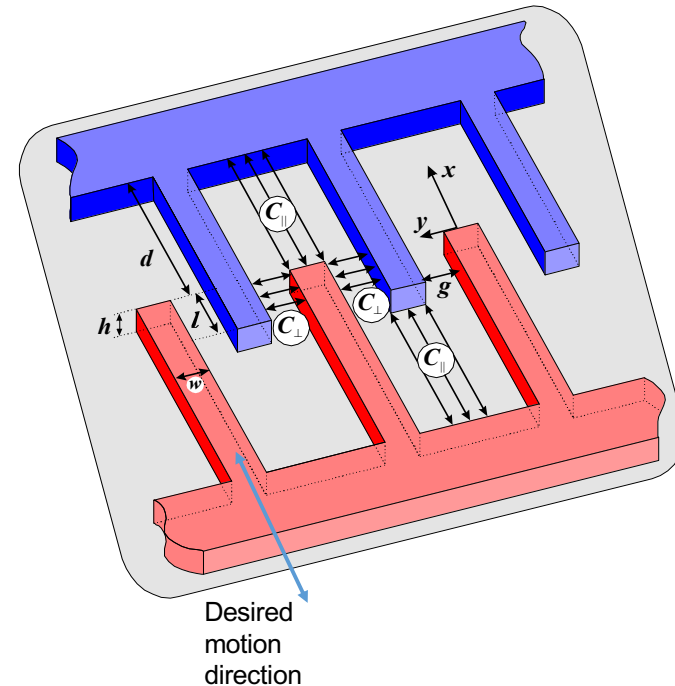


Capacitance $C(l) = 2N\epsilon_0 \left(\frac{l \cdot h}{g} + C_{par} \right)$

Longitudinal sensitivity: $\frac{dC}{dx} \cong N \frac{2\epsilon_0 h}{g}$

⇒ does not depend on total overlap!!

⇒ No x dependence, so no instability



l : overlap length
 h : height or thickness
 g : gap
 N : number of combs

Comb drive actuator

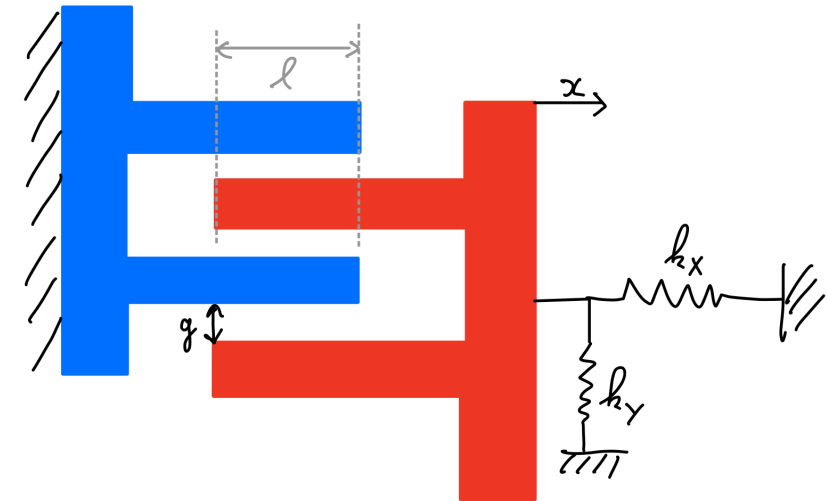
Axial force

$$F_x = \frac{1}{2} \frac{dC}{dx} V^2 = N \cdot \frac{\epsilon_0 h}{g} V^2$$

No x dependence!

Equilibrium Position

$$x = \frac{N \epsilon_0 h V^2}{k_x g}$$



Axial electrostatic Force is the same regardless of overlap! Depends on V^2

Effective spring constant = k_x because F_{es} has no x dependence

No spring softening for comb drive

(but if pull too far, k_x will not be linear, probably get parasitic y motion from flexure support)

Comb drive actuators

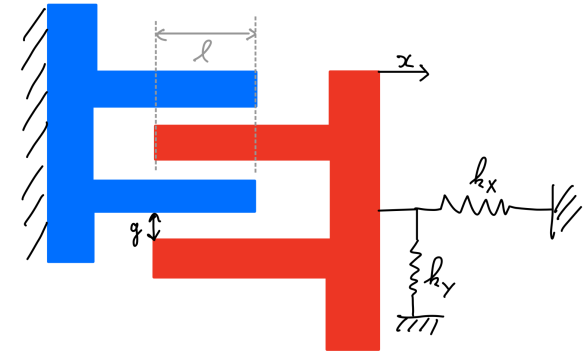
- Axial force is independent of overlap length
- The electrostatic force does not depend on displacement x
=> linear (no spring softening)
- Comb drive allow large displacement, but resonance frequency is limited by the spring mechanism
- Often smaller forces than parallel plate actuators (larger gap)
- The force can be increased by using high aspect-ratio structures: $h > 100 \mu\text{m}$ for SOI

Comb drive actuator

Lateral force

$$F_y = \frac{1}{2} N \varepsilon_0 V^2 h l \left(\frac{1}{(g-y)^2} - \frac{1}{(g+y)^2} \right)$$

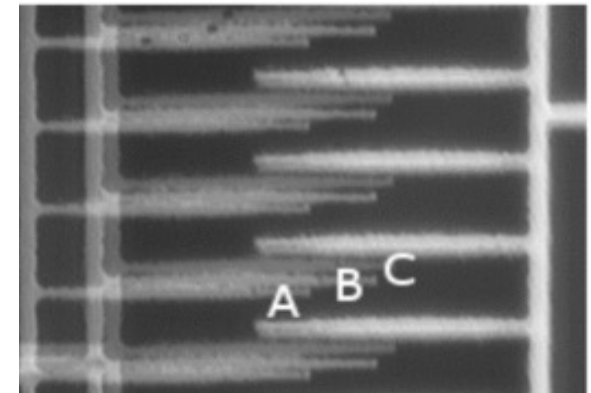
= 0 but only as long as y=0 ...



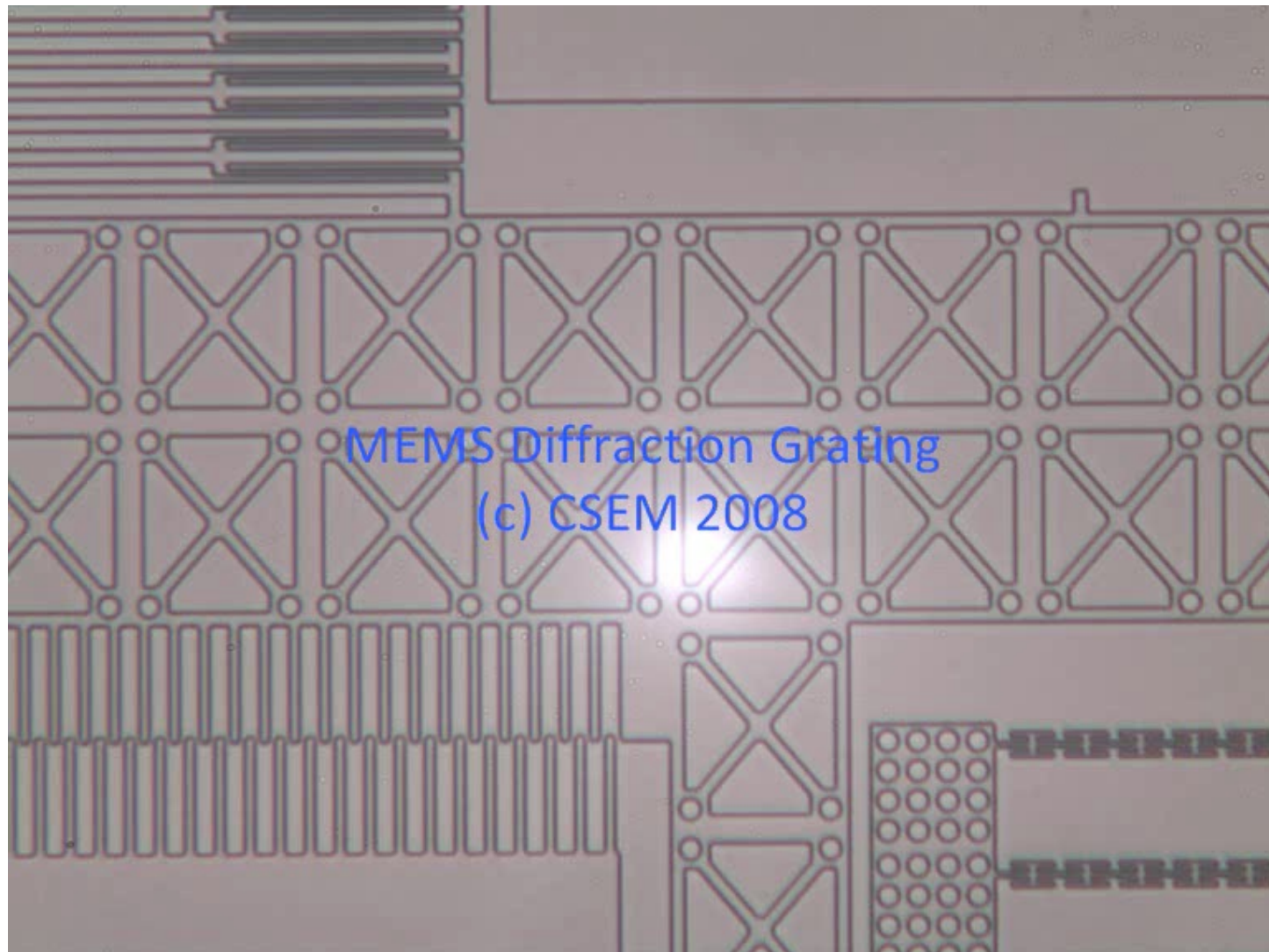
Transverse stability (lateral pull-in)

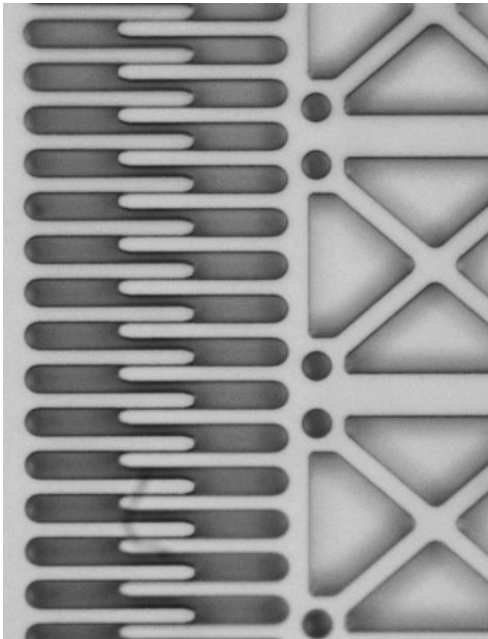
Stability condition (positive spring constant)

$$k_y > \left. \frac{dF_y}{dy} \right|_{y=0} \quad \text{or} \quad \frac{k_y}{k_x} > 2 \frac{x_{\max} (l + x_{\max})}{g^2}$$

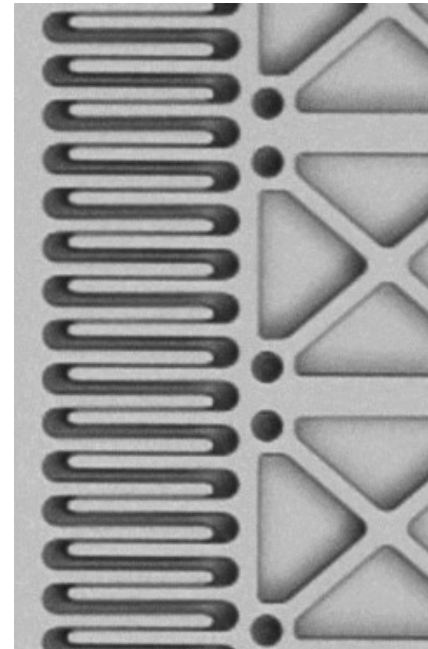


“The lateral instability problem in electrostatic comb drive actuators: modeling and feedback control”
B Borovic et al, (2006) *Journal of Micromechanics and Microengineering*, Volume 16, Number 7





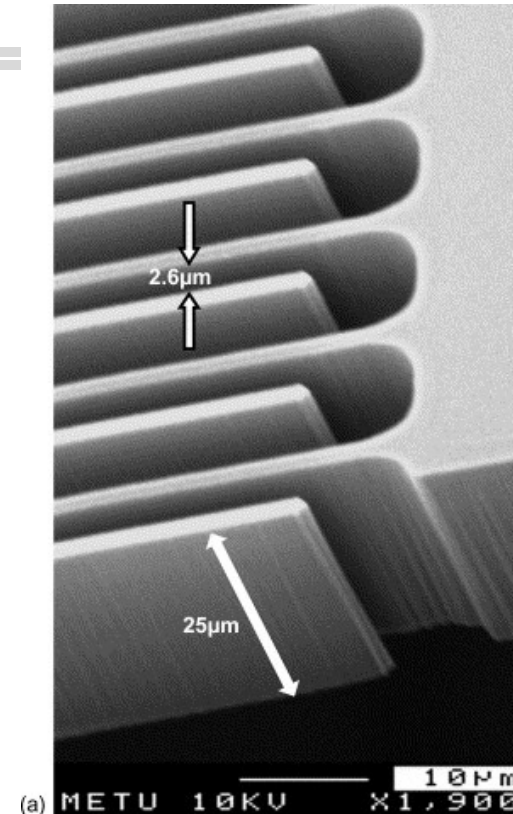
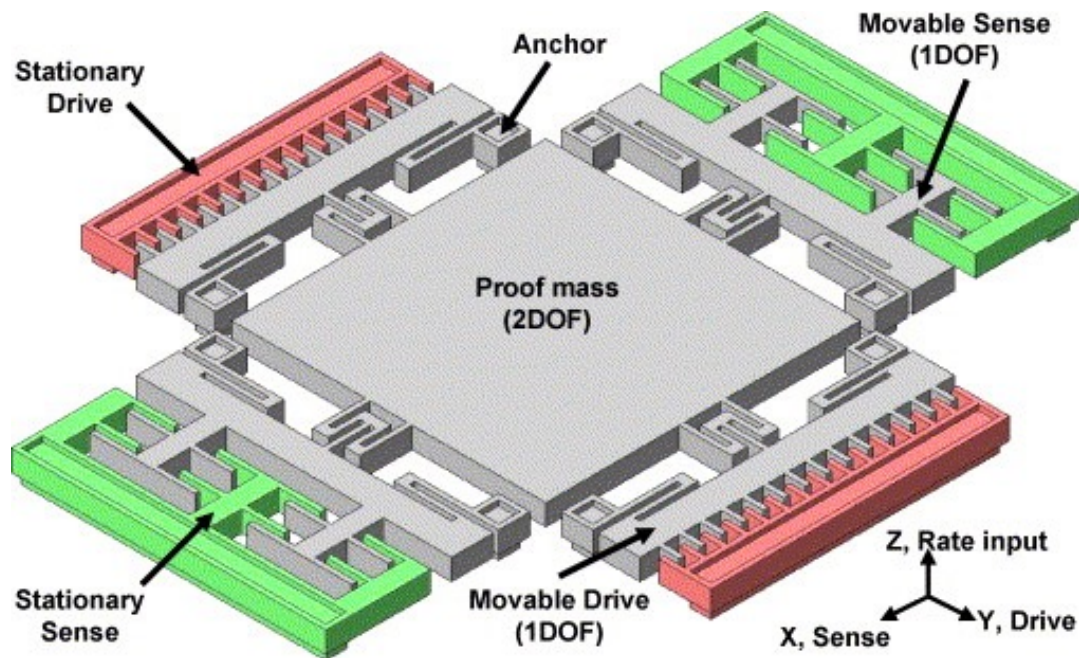
Typical overlap at snap-in



Lateral snap-in with shock

S. Sundaram, et al, *Journal of Micromechanics and Microengineering*, 21, p.045022 (2011)

Example of a MEMS gyro



Typical 1:10 aspect ratio for gap /depth
(SOIMUMPS)

<http://www.memscap.com/products/mumps/soimumps/>

Alper, et al. (2007). "A high-performance silicon-on-insulator MEMS gyroscope operating at atmospheric pressure". *Sensors and Actuators, A: Physical*, 135(1), 34.
<http://doi.org/10.1016/j.sna.2006.06.043>

Electrostatic spring softening example

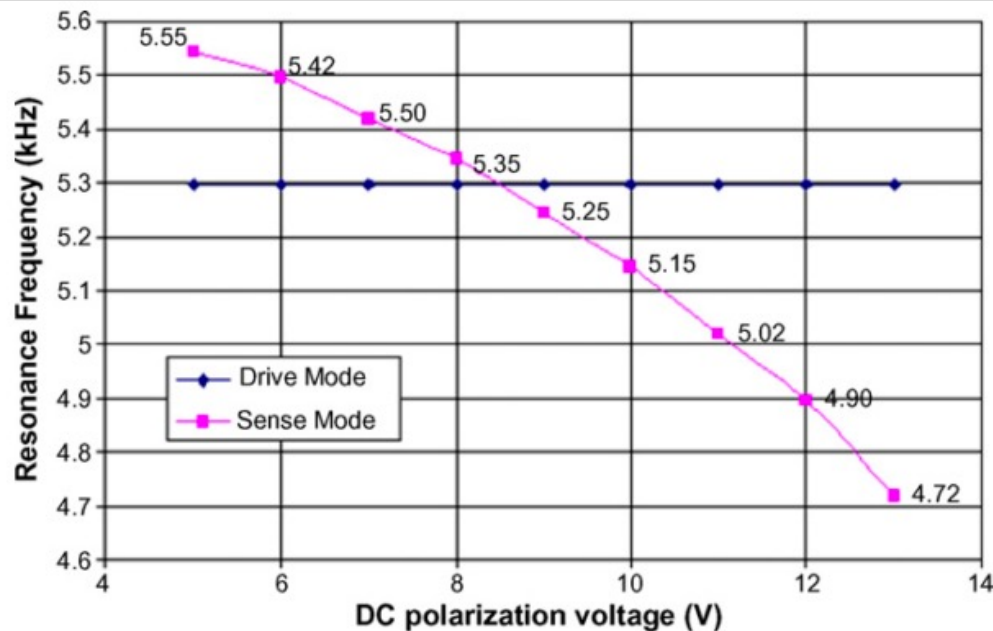


Fig. 8. Demonstration of effective frequency tuning for the sense-mode of the SOI gyroscope. The sense-mode resonance frequency of the gyroscope can be reduced from 5.55 kHz at 5 V dc down to 4.72 kHz at 13 V dc by negative electrostatic spring constant.

For actuation comb drive

$Q = 460$

$k = 140 \text{ N/m}$

$F_{\text{res}} = 5300 \text{ Hz}$

Capacitance: 1000 fF

Finger width: 2 μm

Gap: 2.6 μm

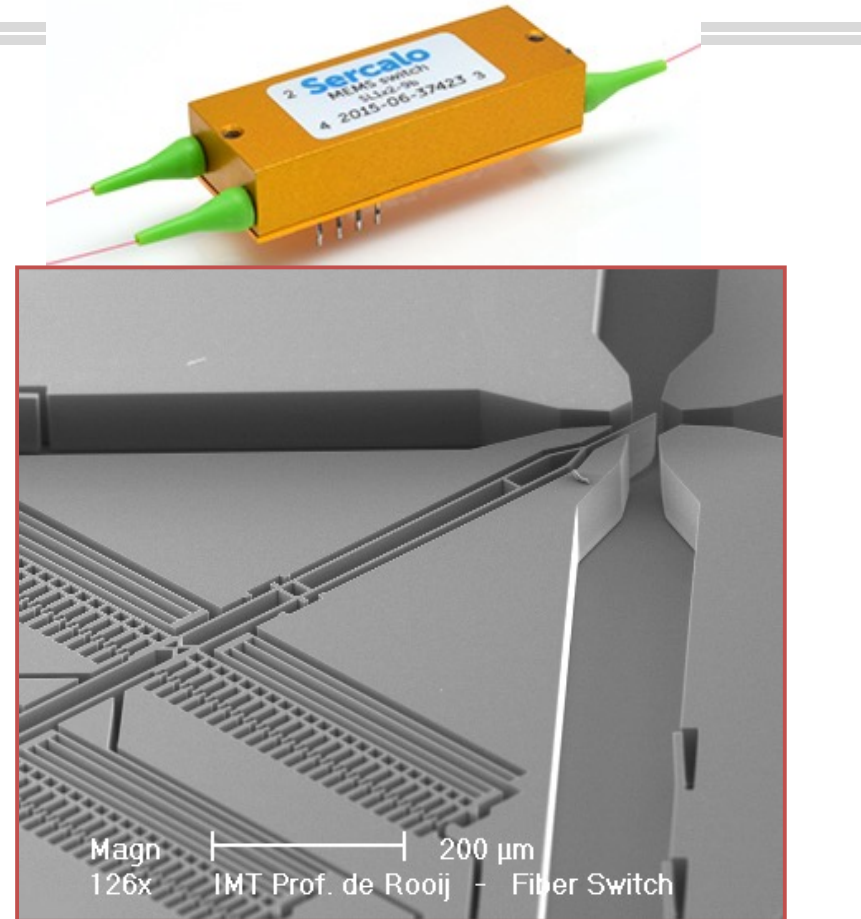
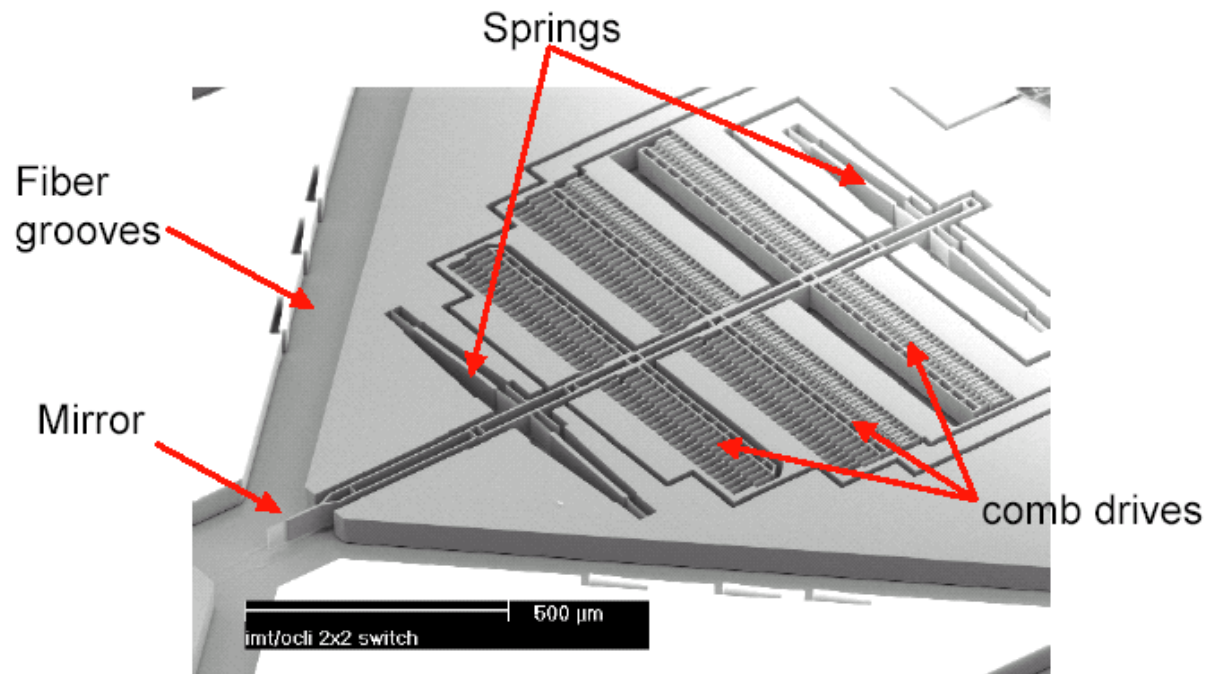
Thickness: 25 μm

Amplitude of Motion = 19 μm

Alper, et al. (2007). "A high-performance silicon-on-insulator MEMS gyroscope operating at atmospheric pressure". *Sensors and Actuators, A: Physical*, 135(1), 34.
<http://doi.org/10.1016/j.sna.2006.06.043>

Comb drive ("drive mode") = no spring softening
 Parallel plates ("sense mode") = significant spring softening

Bistable spring + comb drive for optical switch (Sercalo.com)



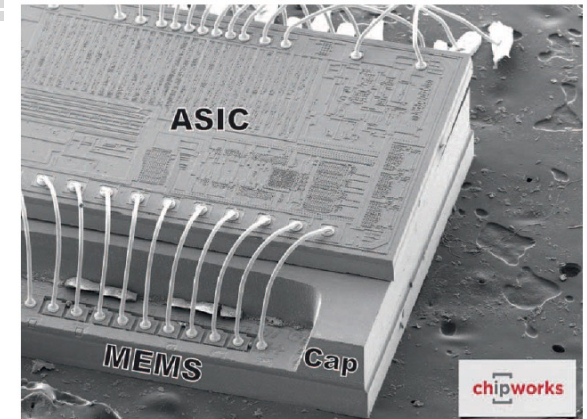
C. Marxer & de Rooij, IMT (www.sercalo.com)

Marxer, C. R., Griss, P., & De Rooij, N. F. (1999). A Variable Optical Attenuator Based on Silicon Micromechanics. *IEEE Photonics Technology Letters*, 11(2), 233–235. <http://doi.org/10.1109/68.740714>

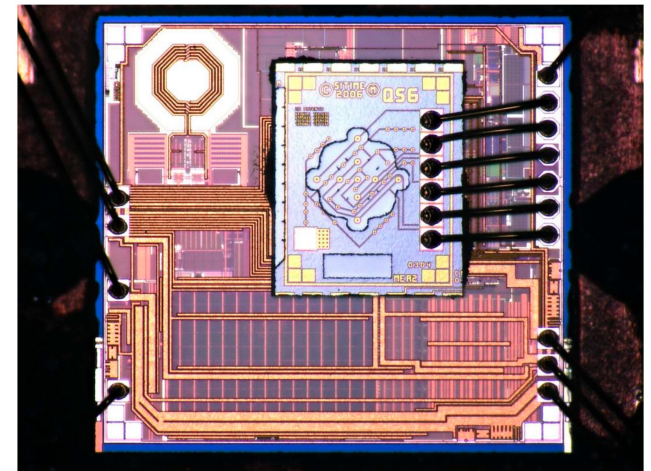
CMOS integration: MEMS first, or MEMS last? Or hybrid

- A. MEMS first, then CMOS (eg early Analog devices accelerometers)
- B. CMOS, then MEMS (eg TI DMD)
- C. **Separate MEMS and CMOS processes, then bond** (most current devices)

Fischer, A. C. *et al.* Integrating MEMS and ICs.
Microsystems & Nanoengineering 1, 15005 (2015).

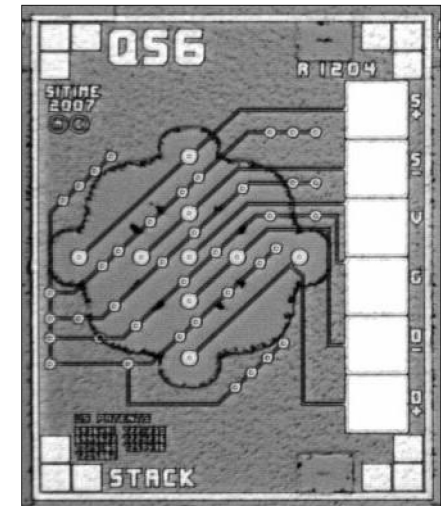
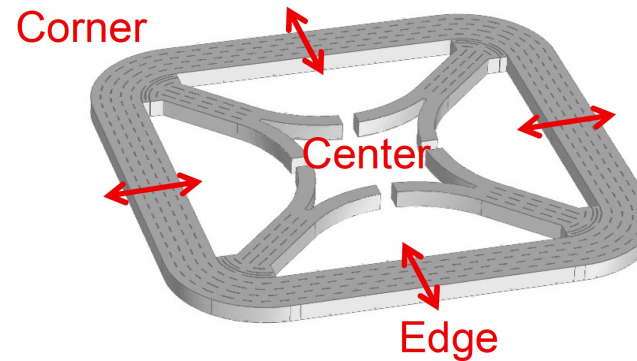
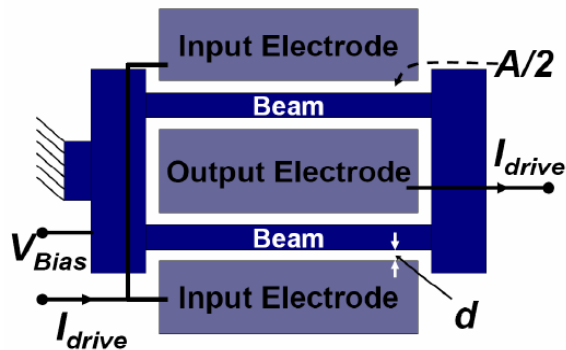
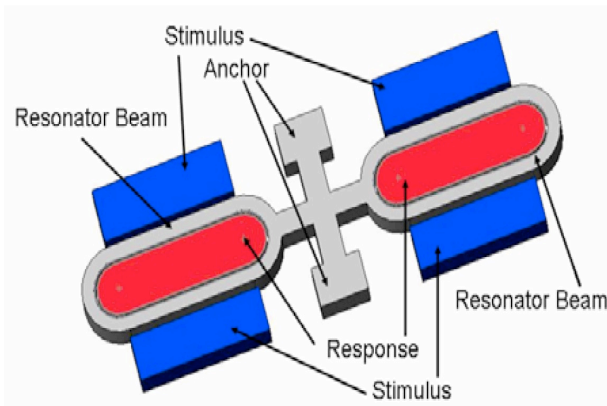


STMicroelectronics LIS331DLH 3-axis accelerometer



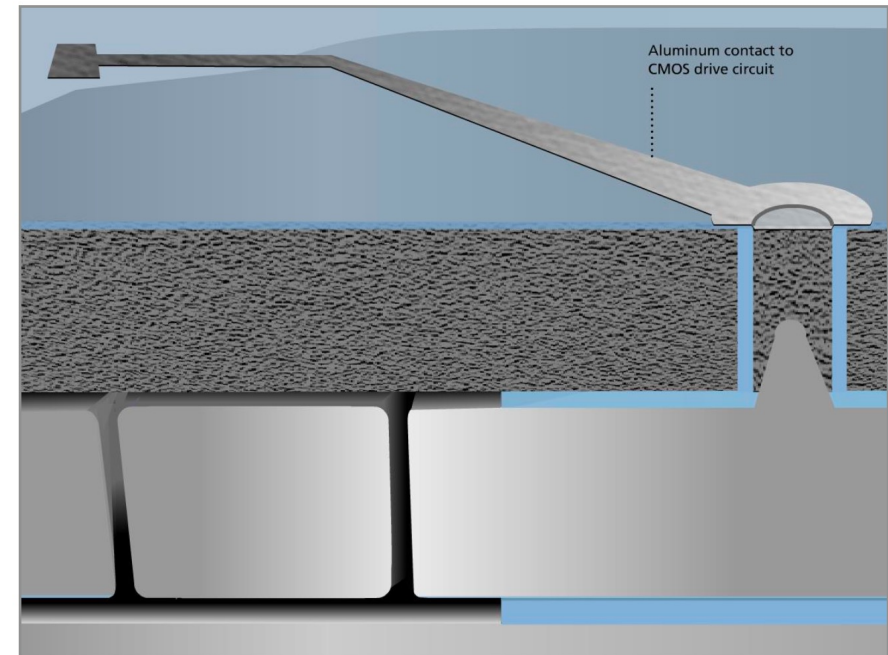
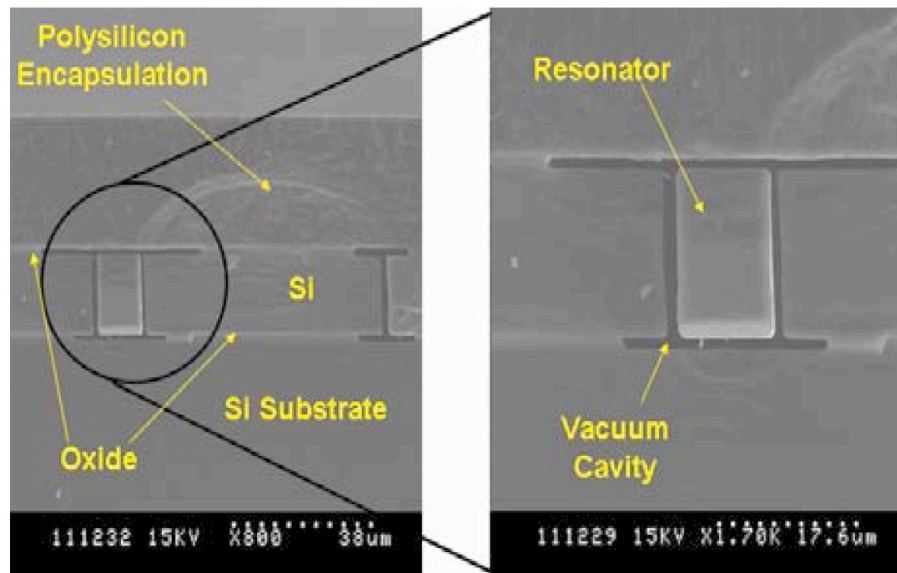
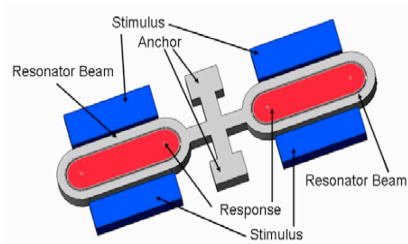
SiTime

SiTime resonators



- Like a 2D bell – held in the center with its outer edges ringing. Motion is a few nanometers.
- Buried inside the chip, not on the surface.

Wafer-scale packaging of resonators



<https://www.sitime.com/sites/default/files/gated/AN20001-MEMS-First-and-EpiSeal-Processes.pdf>

SiTime™

M Agarwal, et al, "Nonlinear Characterization of Electrostatic MEMS Resonators", International Frequency Control Symposium and Exposition, 2006